LEEDS PHILOSOPHICAL & LITERARY SOC

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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E. B. KNOBEL, Esq., President, in the Chair.

Charles Anthony, jun., M.Inst.C.E., Casilla 1045, Buenos Aires, Argentine Republic ;
Henry Osmund Barnard, Superintendent, Trigonometrical Surveys, Ceylon ;
Archibald Young Gipps Campbell, I.C.S., Nandiyal Railway Station P.O., Kurnool District, South India ;
Charles Rundle Davidson, Royal Observatory, and 41 Park Street, Greenwich, S.E. ;
Frank C. Dumat, Johannesburg, Transvaal, South Africa ;
Ambrose T. Flagg, M.A., Chapel House, Westoe, South Shields ;
Walter Heath, M.A., Redcott, Cobham, Surrey ;
John Charles William Herschel, B.A. Oxon, St. John's College, Cambridge, and Lawn Upton, Littlemore, near Oxford ;
James Netherclift Jutsum, Cardiff Nautical Academy, 47 Fitzhamon Embankment, Cardiff, South Wales ;
Captain Joseph W. Martyr, 1 The Glen, South Road, Forest Hill, S.E. ;
Alfred Ernest Moore, B.A., B.Sc. (Lond.), St. John's College, Battersea, S.W. ;
Thomas Marginson Nightingale, B.Sc. (Vict.), 375 Bridgman Street, Bolton, Lancashire ; and
Richard Welford, M.A., J.P., Thornfield, Gosforth, near Newcastle-on-Tyne,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Robert William Chapman, M.A., B.C.E., Lecturer on Engineering and Physical Science in the University, Adelaide, South Australia (proposed by Sir C. Todd) ; and

Charles J. Isaacs, Head of the Upper Nautical School, Greenwich, S.E. (proposed by Thomas Lewis).

Fifty-three presents were announced as having been received since the last meeting, including, amongst others :—

Greenwich Second Ten-year Catalogue of 6892 stars for the epoch 1890·0, presented by the Observatory ; Series of twenty-six Charts of Borneo, Sumatra, &c., for the use of observers of the total solar eclipse of May 1901, presented by Professor Bakhuyzen.

On mechanically compensating the rotation of the field of a Siderostat. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In the *Astrophysical Journal* for March 1900, M. Cornu has given an elegant investigation of the rotation of the siderostat field. To the geometry of his paper little or nothing can be added ; but he has left untouched the question of any mechanical compensation of the variable rotation described, of which I propose to treat in the present note. Some simple link-work arrangements are suggested which it is hoped may be found to solve the problem of compensation practically.

2. Let P (in fig. 1) denote the pole of the heavens ; T the direction in which the telescope is directed, looking from mirror through the OG to the eye end ; and draw the meridian PT, which is not necessarily the meridian of the zenith but that of the telescope.

Consider what happens as a star S passes from upper culmination (on the *telescope* meridian) at S_1 to lower culmination at S_2. If the siderostat reflects its image in the constant direction T, then at upper culmination the normal to the mirror is directed towards N_1 where N_1 bisects the arc TS_1. The pole P will thus be reflected to R_1 *below* T ; and since $PN_1 = N_1R_1$, we have $TR_1 = PS_1$. Similarly at lower culmination the pole is reflected to R_2. And it is easily seen geometrically, as is shown in

M. Cornu's paper, that the image of P describes a circle about T, with radius TR equal to the radius PS, of the circle described by the star about P. The essential difference between the description of the two equal circles is that S moves round P uniformly, but R does *not* move round T uniformly.

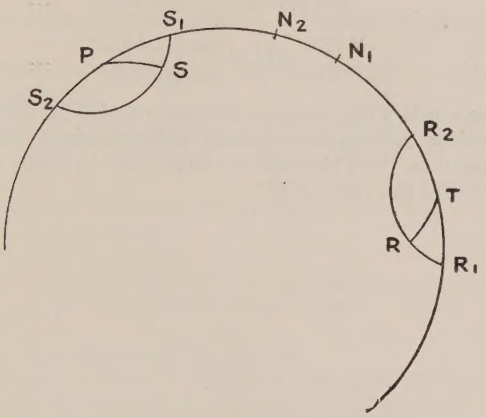

FIG. 1.

3. Its motion is given by the equation

$$\tan \tfrac{1}{2}\mathrm{RTR}_1 = \mathrm{K} \tan \tfrac{1}{2}\mathrm{SPS}_1 \quad \ldots \quad \ldots \quad (1)$$

where K is a constant depending simply on the distances PS and PT. If PS$=\delta$ and PT$=\rho$, then M. Cornu shows* that

$$\mathrm{K} = \frac{\cos \tfrac{1}{2}(\rho+\delta)}{\cos \tfrac{1}{2}(\rho-\delta)} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Thus when $\mathrm{SPS}_1 = 0°$ or $180°$, $\mathrm{RTR}_1 = 0°$ or $180°$, but for intermediate values the angles differ. [For the cœlostat $\rho+\delta = 180°$; hence K$=0$ and $\mathrm{RTR}_1 = 0°$ permanently, except when $\mathrm{SPS}_1 = 180°$. This case is more particularly referred to in § 12.]

4. Now the motion of TR defines the rotational motion required. If we suppose a photographic plate pivoted at T, and moving round T as if TR were rigidly attached to the plate, then all the rest of the picture will remain fixed relatively to the plate, which is the condition required for photography. We proceed to consider various mechanical ways of securing this motion.

5. *First Device.*—If S and C be fixed points, and PN a fixed line at right angles to them on which the point P moves, then

* M. Cornu finds the reciprocal of this expression; but I think there is an algebraical slip in passing from one equation to the other on p. 151 *loc. cit.*

$$\tan \mathrm{PSN} = \frac{\mathrm{PN}}{\mathrm{SN}}$$

$$= \frac{\mathrm{CN}}{\mathrm{SN}} \cdot \frac{\mathrm{PN}}{\mathrm{CN}}$$

or $$\tan \mathrm{PSN} = \frac{\mathrm{CN}}{\mathrm{SN}} \tan \mathrm{PCN} \quad \ldots \quad \ldots \quad (3)$$

Thus, if $$\frac{\mathrm{CN}}{\mathrm{SN}} = \mathrm{K} = \frac{\cos \frac{1}{2}(\rho+\delta)}{\cos \frac{1}{2}(\rho-\delta)} \quad \ldots \quad \ldots \quad (4)$$

and if the angle PCN is made the same as $\frac{1}{2}\mathrm{SPS}_1$ in equation (1), and therefore increases uniformly with the time, then PSN will be the same as $\frac{1}{2}\mathrm{RTR}_1$, and we have only to make another line move at twice the rate of SP.

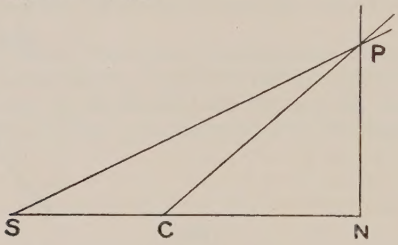

FIG. 2.

Mechanically, SCN and NP must be two bars, PN being grooved. CP and SP must be bars pivoted at S and C, and also grooved; and a pin, P, must connect all three together. CP is to be rotated uniformly by clockwork once in two days, and then SP will move nearly as required. To complete the apparatus we must make another bar or wheel move just twice as fast as SP, which can be done in a variety of ways—*e.g.* by cog-wheel gearing in the ratio of 2 to 1.

6. *Second Device.*—The equation

$$\tan \theta = \mathrm{K} \tan nt \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

is really a particular case of epicyclical motion. Let a point A (fig. 3) be revolving uniformly round a point O at unit distance from it, so that its coordinates may be written

$$x = \cos u \qquad y = \sin u.$$

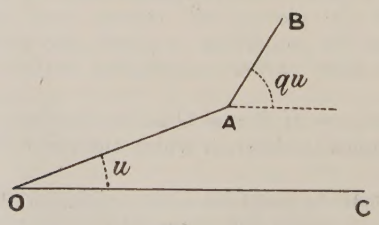

FIG. 3.

Let a second point, B, be revolving uniformly about A, so that its coordinates *referred to* A are

$$x' = p\cos qu \qquad y' = p\sin qu.$$

Then the coordinates of B with respect to O will be

$$x + x' = \cos u + p\cos qu \qquad y + y' = \sin u + p\sin qu\ ;$$

and the direction θ in which B is seen from O is given by the familiar equation

$$\tan\theta = \frac{y+y'}{x+x'} = \frac{\sin u + p\sin qu}{\cos u + p\cos qu} \ldots \qquad \ldots \quad (6)$$

This is the general equation for epicyclical motion.

7. If $q = -1$, we have

$$\tan\theta = \frac{\sin u - p\sin u}{\cos u + p\cos u} = \frac{1-p}{1+p}\tan u \qquad \ldots \quad (7)$$

which resembles equations (1), (3), (5).

In this case the two revolutions are performed in the same period in opposite directions, and the curve is easily seen to be an ellipse.

For the coordinates of B are

$$x + x' = (1+p)\cos u$$

and

$$y + y' = (1-p)\sin u,$$

which represent an ellipse, axes $2(1+p)$ and $2(1-p)$.

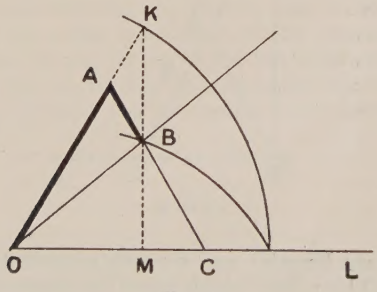

Fig. 4.

Thus let OA, AC be two equal bars, and let OA describe a circle round O uniformly, and let AC be pivoted at A, and the point C constrained to move along the line OL, so that OAC is an isosceles triangle, and AC revolves round A uniformly at the same pace as AO round O, but in the opposite direction. Then any point B on AC, where

$$AB = p\ .\ OA,$$

describes an ellipse ; and further, if a bar, OB, be freely pivoted at O and constrained to pass through B, it will move so that

$$\tan \mathrm{BOL} = \frac{1-p}{1+p} \tan \mathrm{AOL} \quad \ldots \quad \ldots \quad (8)$$

It is easily seen that if OA be produced to K, where

$$\mathrm{AK} = \mathrm{AB},$$

then K lies on the ordinate MB, and also on the auxiliary circle ; and equation (8) is only another form of the well-known relation

$$\mathrm{KM} : \mathrm{BM} = 1+p : 1-p,$$

i.e. in the ratio of the semi-axes.

8. These properties indicate a simple form of ellipsograph, which is probably not new,* but which I do not happen to have seen. It is, however, closely related to an ordinary form. If OL and ON be two fixed bars at right angles, and CD be a bar of constant length whose ends, C, D, slide on OL and ON respectively, then any point B on CD is known to describe an ellipse. If A be the middle point of CD, then OA=AC=AD,

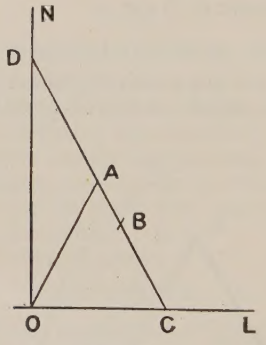

FIG. 5.

and is of constant length ; and we may remove the portion AD if we tie A to O by a bar, which gives the above construction.

9. Having secured the motion of the bar OB so as to satisfy the equation

$$\tan \mathrm{BOL} = \frac{1-p}{1+p} \tan \mathrm{AOC},$$

it remains only to make the bar AO revolve once in two days, as

* Mr. Wesley, after the reading of this paper, drew my attention to the fact that this form of ellipsograph has been described by Mr. Burnham in *Popular Astronomy*, vol. iv. p. 181.

in the first device, and to attach a cog-wheel or other arrangement to OA, so as to double its angular motion. The advantage of this arrangement over the first is that all the bars remain of constant length. In fig. 2 it will be seen that as P travels up NP the bars SP and CP must be longer and longer; and indeed *complete* motion with the first arrangement is impossible. Since we should in practice, however, usually be working near the meridian of the telescope, the first device might be found useful in spite of its disadvantages.

10. *Third Device.*—But there is a third arrangement which is prettier (geometrically) than either. In both the former devices we have had to multiply the movement by 2, by a pair of cog-wheels or some other multiplying arrangement. In fact, we have considered the equation

$$\tan\theta = \mathrm{K}\tan nt \qquad \ldots \qquad \ldots \qquad (9)$$

rather than the equation

$$\tan\tfrac{1}{2}\theta = \mathrm{K}\tan\tfrac{1}{2}nt \qquad \ldots \qquad \ldots \qquad (10)$$

But equation (10) is a familiar one. It represents motion about the focus of an ellipse just as equation (9) represents motion about the centre. In "Dynamics of a Particle" we have the equation

$$\tan\tfrac{1}{2}\theta = \sqrt{\frac{1-e}{1+e}}\,.\,\tan\tfrac{1}{2}u \qquad \ldots \qquad (11)$$

where u is the eccentric anomaly, and θ the true anomaly. Thus in fig. 6 let, as before, two equal bars OA, AC be pivoted at O and A, and let C be free to slide along SOL. Then any point B in AC will describe an ellipse. Let S be the focus of this ellipse, and let a bar pivoted at S be constrained to pass through B. Then it will move in the required manner *without* any multiplying device, if OA revolve uniformly *once* a day. I think this seems the best way of mechanically compensating the rotation.

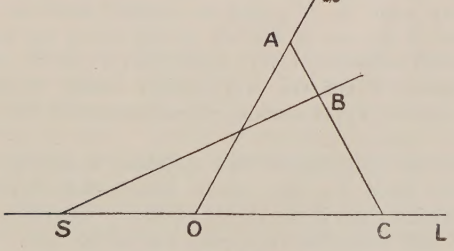

Fig. 6.

11. As regards details, since the unit of length is arbitrary, let us first consider keeping the points O and S fixed. There is

a convenience about this, since S is practically the centre of the plate. The pivot at A must then be altered, and the length of AC adjusted, as well as the point B, to suit the particular star. It seems better to keep OA and AC fixed, and to adjust B and S. To find their positions we have

$$\sqrt{\frac{1-e}{1+e}} = K = \frac{\cos \tfrac{1}{2}(\alpha+\delta)}{\cos \tfrac{1}{2}(\rho-\delta)} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

$$\therefore \frac{1-e}{1+e} = \frac{(1+\cos\rho\cos\delta)-\sin\rho\sin\delta}{(1+\cos\rho\cos\delta)+\sin\rho\sin\delta}$$

or
$$e = \frac{\sin\rho\sin\delta}{1+\cos\rho\cos\delta} \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

Further, if AB$=p$. AO, the axes are $1+p$ and $1-p$; or

$$(1+p)^2(1-e^2) = (1-p)^2$$

or
$$e^2 = \frac{4p}{(1+p)^2} \quad \ldots \quad \ldots \quad (14)$$

Hence, $p = 2-e^2+2\sqrt{1-e^2} = (1+\sqrt{1-e^2})^2$ ... ... (15)

and OS$= e(1+p) = 2\sqrt{p} = 2+2\sqrt{1-e^2}$ ... ... (16)

12. *The Cœlostat.*—The particular case of the Cœlostat deserves notice. In § 2 it was remarked that the reflected image R of the Pole always describes a circle about T of radius equal to PS—*i.e.* of finite radius, depending only on the star. And yet in the Cœlostat we know that the image of the Pole, like every other point of the sky, remains fixed. How is this apparent paradox to be explained? The above construction shows the way in which it occurs as a limiting case. The limit is when $\cos\frac{1}{2}(\rho+\delta)=0$, and $e=1$. The ellipse thus becomes a parabola, and S is at an infinite distance.

But suppose that, as suggested in the beginning of the last paragraph, we keep the length OS constant, and vary OA and AC. As e approaches unity, p approaches unity—*i.e.* B approaches very near to C ; and its motion becomes nearly the same as that of C—*i.e.* it travels nearly along the straight line LO. Thus SB remains nearly coincident with SC, *until* B gets near S. Then SB travels very swiftly round through nearly 360°, and comes to approximate coincidence with SC, but on the other side.

Thus the limiting case of the cœlostat is one in which the plate remains fixed for the whole twenty-four hours less one instant, in which time it suddenly makes a complete revolution. The arc TR still describes its circle, but in *no time.*

It will now be clear how the cœlostat occupies the position of transition between cases where TR describes its circle in opposite directions. The revolution in *no time*—*i.e.* with infinite velocity—may be performed indifferently in either direction

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just as asymptotes meet a curve at infinity in either direction. If this explanation is correct, the case is a curiosity which has (so far as I know) hitherto escaped attention.

Observations of Saturn made at Juvisy Observatory in 1900.
By C. Flammarion.

The planet was observed from July to October last, and, although the number of clear nights was abnormally great, definition was seldom satisfactory.

The instrument employed was the $9\frac{3}{4}$-inch equatorial, bearing negative eyepieces magnifying 218, 300, 400, and 600 diameters, the observers being M. Antoniadi and myself.

1. *The Globe.*

The N. polar cap was not very dark in 1900. No certain traces of the N. temperate band were seen. The great double tropical belt was, however, a very striking feature of the planet, its duplex character being recognised under almost any kind of definition—even when the Cassini division was invisible.* The dark spots occasionally found on this belt were much better seen in 1900 than in 1899. Thus, one of them, having a condensation on each of the components of the belt, was seen in transit over the central meridian on 1900 July $15^d\ 11^h\ 10^m$ G.M.T. Another, much more marked spot, but having no condensation on the N. branch, was central on September $5^d\ 7^h\ 41^m$; and a third, double spot, on September $5^d\ 8^h\ 33^m$. By far the brightest, though yellowest, portion of the globe was the equatorial zone. It was very uneven in tint, and mottled in appearance. A faint dusky band, almost marking the equator, was seen with certainty on July 10 and September 5.

The decreasing luminosity of the globe towards the limb was very marked.

2. *The Rings.*

(*a*) *Outer Ring, A.*—Encke's division could never be seen, though carefully looked for. The indentations along the ansæ, near Cassini's division, were easily seen, and have been noted on many occasions in 1900. As far as could be judged, their structure was triangular; one side of each triangle resting on the inner edge of this Ring.

As usual, Cassini's division seemed grey and not black. Probably there is some matter in it. In fact, Maxwell has shown that the collisions among the particles would tend to widen the several rings. The division seemed still tangent to the globe,

* Dr. Deslandres was recently successful in photographing the double belt with the great 23·6-inch photographic equatorial of Meudon.

though the presence of the planet's shadow rendered this observation rather difficult.

(*b*) *Inner Bright Ring, B.*—This has been invariably destitute of detail, the gradations shown by Trouvelot being quite invisible in 1900, though they have been so marked in the last few years. But the inner edge of B was exceedingly indefinite, and at times it was impossible to say where the bright ring ended and where the "dark" began.

(*c*) *The "Crape" Ring, C.*—This was fainter than usual along the ansæ, an effect due, very likely, to the low altitude of the planet. On October 2 the appearance described on pp. 21–22 of the present volume, and now given in the annexed fig. 1, was

FIG. 1. Appearance of the "Crape" Ring on 1900 October 2.

detected easily, when the part of the "Crape" Ring across the globe did not seem to have its inner edge in continuation of the part projected on the sky.

The shadow of the planet had an irregular outline owing to the varying luminosity of the Ring (fig. 2). Irradiation caused

FIG. 2. Irregularity in the planet's shadow, due to irradiation.

it to appear to turn round in a sharp curve into the Cassini division.

Lastly, as in previous years, the eastern vacuity between *Saturn* and the Ring seemed greater than the western, the effect being very marked indeed in 1900.

Note on the Rotation Period of Saturn in 1896 *and* 1897.

By C. Flammarion.

Systematic observations of *Saturn* were started here in 1894, and since that time the work has been carried on to the present year.

In 1894 very few details were seen on the globe, and the double structure of the great dusky belt north of the equatorial zone was scarcely recognised. No spots, dark or white, were seen at that time.

In 1895 the planet was seen much better. The duplicity of the north equatorial belt was well marked, while the narrow band marking the equator was also an obvious feature. On 1895 July 8 a very small dark spot was seen on this band, transiting the central meridian at 9^h G.M.T. This marking was well suited for getting a very accurate determination of the rotation period of the material very near the equator; but it was never seen again.

On 1896 June 27 M. Antoniadi, by stopping down the aperture of the $9\frac{3}{4}$ inch equatorial to $6\frac{1}{2}$ inches, detected a faint dusky spot on the great belt; it was double, having a condensation on each of the two components of the belt. A fainter single spot was seen meantime preceding by a few minutes the double spot. Since that time the dark spots of *Saturn* were easily seen whenever the atmospheric conditions were not very unfavourable.

A thorough discussion of the observations gives the following results :—

Three of the dark spots seen in 1896 could be followed through several transits over the central meridian.

Spot.	First seen.	Last seen.	No. of Rotations.	Rotation Period. h	m	s
1896 A	June 27	July 6	21	10	13	58·0
„ B	„ 27	„ 11	33	10	14	39·6
„ C	„ 29	„ 11	28	10	14	4·1
	Mean period of rotation			10	14	14·

In 1897 the dusky spots were very easy objects, and a great number of them have been seen by myself and other observers.

Four spots could be followed through more than one transit.

Spot.	First seen.	Last seen.	No. of Rotations.	Rotation Period. h	m	s
1897 A	June 13	August 19	157 *	10	14	0·6
„ B	July 10	July 18	19	10	13	23·3
„ C	„ 10	„ 16	14	10	14	15·2
„ D	„ 10	„ 16	14	10	14	36·6
	Mean period of rotation			10	14	4·

It became increasingly difficult to see the spots, as the southern declination of *Saturn* increased in 1898, 1899, and 1900, and the rarity with which they were seen stood in the way of any serious identification of them. The results, however, for 1896 and 1897 are satisfactory for +18° of kronocentric latitude, and in fair agreement with the work of other observers.

On the Accuracy of Eye-observations of Meteors and the Determination of their Radiant Point. By Bryan Cookson.

The probable existence of stationary radiants for so many meteoric showers and the difficulty of finding a satisfactory physical explanation of them, makes the question of the determination of the radiant of considerable importance. The following is an attempt to obtain a numerical estimate of the accuracy of eye-observations of meteors. So far only *Perseids* have been used for the purpose, this being the shower which afforded the largest number of bright and easily observed meteors ; but it would be interesting to apply the method to other showers with different characteristics, such as those of April and October.

A chart of the celestial sphere on the gnomonic projection was drawn on a scale of about $\frac{1}{10}$-inch to 1° R.A., and $\frac{1}{5}$-inch to 1° Decl., with centre at R.A. 45°, Decl. +57°, which is the assumed position of the *Perseid* radiant on August 10. From the recorded R.A. and Decl. of beginning and end, the paths of the meteors were drawn upon tracing paper placed on the chart, and the equation of the line of the meteor referred to rectangular coordinates, with their origin at R.A. 45°, Decl. +57, was found graphically by an easy and rapid method in the form

$$x \cos a + y \sin a - p = 0.$$

The radiant is the point such that the sum of the squares of the perpendiculars from it to these lines is a minimum. We have, therefore, to solve by least squares a system of equations of the above form for x and y, which are small corrections to be applied to the approximate radiant.

* The identity of the spots seen on 1897 June 13 and August 19 is somewhat doubtful, as there are no intermediate observations.

Weights have been assigned to the equations based upon the following considerations. It is assumed that for a given observer an error in the recorded beginning or end of a meteor's path is equally probable in any direction. This is not, however, strictly true: Professor Weiss found that the probable error of a recorded beginning or end was about $\pm 2°{\cdot}95$, but that the direction of path was more accurate than this large error would lead one to expect. This is owing to the fact that many observers record the end of a trail in the prolongation of the meteor's path, and hence by comparing observations of the same meteors by different observers the differences in the celestial coordinates of the end may be considerable, whilst the direction of the path may be very nearly the same for all. The probable error as far as direction is concerned was found to be $\pm 1°{\cdot}0$ (*Sitzungsber. Wien Akad.* vol. 62 Abt. II. pp. 277–344). As we are here only concerned with direction, it seems legitimate to make the above assumption.

Let then a small arc ρ of great circle be resolved into two perpendicular components: to find the mean error of these components ϵ_1 and ϵ_2, corresponding to a mean error ϵ of ρ, we have

$$\epsilon_1^2 = \frac{\epsilon^2}{2\pi}\int_0^{2\pi} \cos^2\psi d\psi = \frac{\epsilon^2}{2} \quad : \quad \epsilon_2^2 = \frac{\epsilon^2}{2\pi}\int_0^{2\pi} \sin^2\psi d\psi = \frac{\epsilon^2}{2}.$$

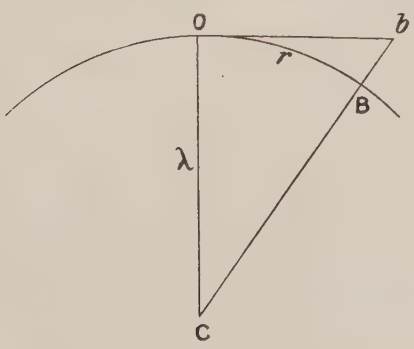
The mean error of the components is therefore $\frac{\epsilon}{\sqrt{2}}$.

If now B be projected into b, the coordinates of b are

$$x = \lambda \tan r \cos\theta$$
$$y = \lambda \tan r \sin\theta.$$

The small displacements of B in perpendicular directions are clearly

$$dr \text{ and } \sin r \,.\, d\theta,$$



and we have just seen that the mean error of each is $\epsilon/\sqrt{2}$.

But it is known that if a linear differential relation such as

$$dP = A\,dx + B\,dy + C\,dz + \ldots$$

holds good, then

$$\epsilon_P^2 = A^2\epsilon_x^2 + B^2\epsilon_y^2 + C^2\epsilon_z^2 + \ldots$$

where ϵ_P denotes the mean error of P, &c. In our case we have

$$dx = \lambda \sec^2 r \cos\theta\, dr - \lambda \tan r \sin\theta \frac{1}{\sin r}(\sin r\, d\vartheta),$$

and therefore

$$\begin{aligned}\epsilon_x^2 &= \lambda^2 \sec^4 r \cos^2\theta \frac{\epsilon^2}{2} + \lambda^2 \tan^2 r \sin^2\theta \frac{1}{\sin^2 r}\frac{\epsilon^2}{2}\\ &= \frac{\epsilon^2}{2}\lambda^2 \sec^2 r\,[(1+\tan^2 r)\cos^2\theta + \sin^2\theta]\\ &= \frac{\epsilon^2}{2}(\lambda^2+x^2+y^2)\left(1+\frac{x^2}{\lambda^2}\right)\\ &= \frac{\epsilon^2}{2}\frac{(\lambda^2+\rho^2)(\lambda^2+x^2)}{\lambda^2}.\end{aligned}$$

Similarly,

$$dy = \lambda \sec^2 r \sin\theta\, dr + \lambda \tan r \cos\theta \frac{1}{\sin r}(\sin r\, d\theta)$$

and

$$\begin{aligned}\epsilon_y^2 &= \frac{\epsilon^2}{2}\lambda^2 \sec^2 r\,[(1+\tan^2 r)\sin^2\theta + \cos^2\theta]\\ &= \frac{\epsilon^2}{2}\frac{(\lambda^2+\rho^2)(\lambda^2+y^2)}{\lambda^2}.\end{aligned}$$

I am indebted to Professor Turner for pointing out this method of deducing the expressions for ϵ_x and ϵ_y; it leads to the same results but is much shorter and more elegant than that originally used by me.

Now, since in the equations of condition $x\cos\alpha + y\sin\alpha - p = 0$ x and y are small quantities, we may neglect the mean errors of $\cos\alpha$ and $\sin\alpha$, and consider the weight of the equation as proportional to the square of the mean error of the absolute term, p.

Let, therefore, x_1y_1 be the coordinates of the beginning of the meteor's path, and x_2y_2 those of the end.

Then

$$p = \frac{y_2x_1 - y_1x_2}{\{x_2 - x_1^2 + y_2 - y_1^2\}^{\frac{1}{2}}} = \frac{y_2x_1 - y_1x_2}{D}$$

$$\therefore\ dp = \frac{1}{D}\left\{y_2 + \frac{p}{D}(x_2 - x_1)\right\}dx_1 - \frac{1}{D}\left\{y_1 + \frac{p}{D}x_2 - x_1\right\}dx_2$$
$$- \frac{1}{D}\left\{x_2 - \frac{p}{D}(y_2 - y_1)\right\}dy_1 + \frac{1}{D}\left\{x_1 - \frac{p}{D}y_2 - y_1\right\}dy_2$$
$$= \frac{1}{D}\left\{y_2 - p\sin\alpha\right\}dx_1 - \frac{1}{D}\left\{y_1 - p\sin\alpha\right\}dx_2$$
$$- \frac{1}{D}\left\{x_2 - p\cos\alpha\right\}dy_1 + \frac{1}{D}\left\{x_1 - p\cos\alpha\right\}dy_2$$
$$= -\frac{1}{D}\sqrt{\rho_2^2 - p^2}\,.\cos\alpha\,.\,dx_1 + \frac{1}{D}\sqrt{\rho_1 - p^2}\,.\cos\alpha\,dx_2$$
$$- \frac{1}{D}\sqrt{\rho_2^2 - p^2}\ \sin\alpha\,.\,dy_1 + \frac{1}{D}\sqrt{\rho^2 - p^2}\,.\,\sin\alpha\,.\,dy_2.$$

Hence,

$$\varepsilon_p^2 = \frac{1}{D^2}\left[\cos^2\alpha\left\{(\rho_2^2 - p^2)\varepsilon^2_{x_1} + (\rho_1^2 - p^2)\varepsilon^2_{x_2}\right\} + \sin^2\alpha\left\{(\rho_2^2 - p^2)\varepsilon^2_{y_1} + (\rho_1^2 - p^2)\varepsilon^2_{y_2}\right\}\right]$$

or

$$\varepsilon_p^2 = \frac{\varepsilon^2}{2}\frac{1}{\lambda^2 D^2}\left[(\lambda^2 + \rho_1^2)(\rho_2^2 - p^2)(\lambda^2 + x_1^2\cos^2\alpha + y_1^2\sin^2\alpha) + (\lambda^2 + \rho_2^2)(\rho_1^2 - p^2)(\lambda^2 + x_2^2\cos^2\alpha + y_2^2\sin^2\alpha)\right]$$
$$= \frac{\varepsilon^2}{2}\frac{1}{\lambda^2 D^2}\left[(\lambda^2 + p^2)\left\{(\lambda^2 + \rho_1^2)(\rho_2^2 - p^2) + (\lambda^2 + \rho_2^2)(\rho_1^2 - p^2)\right\} - 2\sin\alpha\cos\alpha\left\{x_1 y_1(\lambda^2 + \rho_1^2)(\rho_2^2 - p^2) + x_2 y_2(\lambda^2 + \rho_2^2)(\rho_1^2 - p^2)\right\}\right]$$
$$= \frac{\varepsilon^2}{2}\frac{1}{\lambda^2 D^2}\left[(\lambda^2 + p^2)\mathrm{P} - 2\sin\alpha\cos\alpha\,\mathrm{Q}\right], \text{ suppose.}$$

The scale of projection used is such that $\lambda = 10$ inches; the mean value of p is then about ·5, and of ρ_1 about 3; hence, excepting in a very few cases where ρ_1 is small (< 1), that is when the meteor starts from very near the radiant, it is safe to neglect p^2 and take

$$\mathrm{P} = \rho_1^2(\lambda^2 + \rho_2^2) + \rho_2^2(\lambda^2 + \rho_1^2).$$

If $\theta_1\theta_2$ are the angles which the lines joining the beginning x_1y_1 and end x_2y_2 of the meteor's path make with the axis of x, then

$$\frac{x_1 y_1}{\rho_1^2} = \frac{1}{2}\sin 2\theta_1 \qquad \frac{x_2 y_2}{\rho_2^2} = \frac{1}{2}\sin^2\theta_2$$
$$\therefore\ \frac{x_1 y_1}{\rho_1^2} - \frac{x_2 y_2}{\rho_2^2} = \sin(\theta_1 - \theta_2)\cos(\theta_1 + \theta_2)$$
$$= \left(\frac{y_1}{\rho_1}\frac{x_2}{\rho_2} - \frac{y_2}{\rho_2}\frac{x_1}{\rho_1}\right)\cos(\theta_1 + \theta_2)$$
$$= -\frac{pD}{\rho_1\rho_2}\cos(\theta_1 + \theta_2)$$

Thus

$$Q=x_1y_1\left[(\lambda^2+\rho_1^2)(\rho_2^2-p^2)+\frac{\rho_2^2}{\rho_1^2}(\lambda^2+\rho_2^2)(\rho_1^2-p^2)\right]$$
$$+\frac{pD\rho_2}{\rho_1}\cos(\theta_1+\theta_2)(\lambda^2+\rho_2^2)(\rho_1^2-p^2)$$

In an unfavourable case we might have $x_1=y_1=2$; $p=\frac{1}{2}$, $\rho_1=2\sqrt{2}$, $\rho_2=3\sqrt{2}$, $D=\sqrt{2}$; the ratio of the last two terms in Q is then at most

$$4\,.\,\tfrac{9}{4}:\tfrac{1}{2}\sqrt{2}\,.\,\tfrac{3}{2}\text{ or nearly }8:1.$$

We are therefore justified in neglecting the last term as well as p^2 in the first two terms ; and a table of values of P and Q′ where

$$Q'=\rho_1^2\rho_2^2(2\lambda^2+\rho_1^2+\rho_2^2)$$

was constructed.

The coefficient of Q′ is $-2\sin\alpha\cos\alpha\,\frac{x_1y_1}{\rho_1^2}$ and is always positive ; the value of x_1y_1/ρ_1^2 or $\frac{1}{2}\sin 2\theta_1$ can easily be read off to two decimal places by placing a piece of tracing paper on the chart having lines ruled, along which $\frac{1}{2}\sin 2\theta_1$ has the values 0·1 to 0·5.

The weight of an equation is inversely proportional to the square of the mean error of its absolute term, and in the present case the weight is therefore proportional to

$$\frac{\lambda^2D^2}{\lambda^2P+2\sin\alpha\cos\alpha\,\frac{x_1y_1}{\rho_1^2}Q'}$$

which is the simplest form to which the expression for the weight can be reduced. The numerical value of D is found by means of a foot rule, P and Q′ are taken together out of the table ; x_1y_1/ρ_1^2 is read off graphically, and $2\sin\alpha\cos\alpha$ is twice the product of the coefficients of x and y. No attention has to be paid to signs. For convenience the weights are multiplied by 100.

The table of P and Q′ which was actually used is here given ; the values of Q′ are printed in italics immediately beneath the corresponding values of P.

Let g be the weight of an equation (m in number),

v be a residual—*i.e.* the perpendicular from the deduced radiant on the meteor's path.

Then the mean error of an absolute term whose weight is unity is

$$\sqrt{\frac{[gvv]}{m-2}}.$$

TABLE OF P AND Q′.

$P = \rho_1^2(1+\rho_2^2) + \rho_2^2(1+\rho_1^2)$ and $Q' = \rho_1^2\rho_2^2(1+\rho_1^2+1+\rho_2^2)$

Neglect p when $p < .05$
$\rho_1 > .80$

$p = .06$, subtract .01 from P

$p = .07$, „ .01 „

$p = .08$, „ .01 „

$p = .09$, „ .02 „

$p = .10$, „ .02 „

$p = .11$, „ .02 „

$p = .12$, „ .03 „

$p = .13$, „ .03 „

$p = .14$, „ .04 „

$p = .15$, „ .04 „

$p = .16$, „ .05 „

TABLE OF P AND Q′.

$P = \rho_1^2(1+\rho_2^2) + \rho_2^2(1+\rho_1^2)$ and $Q' = \rho_1^2\rho_2^2(1+\rho_1^2+1+\rho_2^2)$

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$p = .09$, " .02 "

$p = .10$, " .02 "

$p = .11$, " .02 "

$p = .12$, " .03 "

$p = .13$, " .03 "

$p = .14$, " .04 "

$p = .15$, " .04 "

$p = .16$, " .05 "

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The mean error of an absolute term of weight g, or of an observed p, is therefore

$$\frac{1}{\sqrt{g}}\sqrt{\frac{[gvv]}{m-2}}.$$

But we have just found that this was ϵ_p, where

$$\epsilon_p^2=\frac{\epsilon^2}{2}\frac{1}{\lambda^2 D^2}\left\{\lambda^2 P+2\sin\alpha\cos\alpha\,\frac{x_1y_1}{\rho_1^2}Q'\right\}$$

and we have taken

$$g=100\,\frac{\lambda^2D^2}{\lambda^2P^2+2\sin\alpha\cos\alpha\,\frac{x_1y_1}{\rho_1^2}Q}.$$

Hence

$$\frac{1}{\sqrt{g}}\sqrt{\frac{[gvv]}{m-2}}=\epsilon_p=\frac{\epsilon}{\sqrt{2}}\,\frac{10}{\sqrt{g}},\text{ or }\epsilon=\tfrac{1}{10}\sqrt{\frac{2}{m-2}[gvv]}.$$

Now g is of -2 dimensions in length, and ϵ is therefore a number, and is the circular measure of the arc of great circle, which is the mean error of an observed beginning or end of a meteor's path.

The effect of zenith attraction, rotation and orbital motion of the Earth have been neglected as inappreciable. For let ξ, η be the coordinates of the apparent radiant referred to parallel axes through the unknown true radiant; let x, y be the coordinates of the true radiant. Then ξ, η are known at any moment, when the character of the swarm is known. The perpendicular distance from the point $x+\xi$, $y+\eta$ on to the line of a meteor's track is

$$(x+\xi)\cos\alpha+(y+\eta)\sin\alpha-p=0.$$

The error committed in neglecting ξ, η is therefore

$$dp=\xi\cos\alpha+\eta\sin\alpha,$$

of which the maximum value is $\sqrt{\xi^2+\eta^2}$, when $\tan\dot{\alpha}=\eta/\xi$. From Dr. Kleiber's table in *Monthly Notices*, lii. p. 352, we find that, to reduce observations made on a given night to the midnight of that night, we must apply the following corrections :—

	$\Delta\alpha$	$\Delta\delta$	ξ	η	$\sqrt{\xi^2+\eta^2}$
9 P.M.	−0°·40	+0°·71	−·38	+1·24	1·3
Midnight	−0°·71	+0°·49	−·67	+ ·89	1·1
3 A.M.	−0°·63	+0°·24	−·59	+ ·42	0·7

The unit of p was for convenience always taken as $\frac{1}{10}$ inch; x, y, ξ, η are also expressed in terms of this unit. As the probable error of x and y generally amounts to about $\pm 0\cdot5$, it will be quite

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sufficient to apply the mean correction $\xi = -0\cdot5$, $\eta = +1\cdot0$ to the values of x and y, deduced from meteors fairly evenly distributed in time from 9 P.M. to 3 A.M., in order to get the absolute place at midnight. Had these disturbing causes produced appreciable effects, the path of the apparent radiant should be drawn round the origin as true radiant, and p measured from the point on this curve corresponding to the time of observation instead of from the origin. In the case of the *Perseids* this point does not appreciably shift during the night.

The observations discussed so far are :—

		Meteors.	
W. F. Denning at Bristol		149	Communicated in manuscript.
M. Morine	Poulkova	49	"Sur les Perséids, observées en 1890." (*Brédikhine, Bull. St. Pét. Acad.* 1894, No 1.)
M. Maggi	Volpeglino	46	*Pub. Osservatorio di Brera*, No. 7, p. 58.
M. Scharbe	Dorpat	43	*Astr. Nach.* Bd. 149, p. 213.
M. Lewitzky	Dorpat	23	" " " "
	Total ...	310	

Mr. Denning very kindly copied out some of his observations of *Perseids* and sent them to me at my request. I am much indebted to him for having done this.

The results from these observations are as follows :—

Observer.	Date. August.	r. Probable Error of p of weight unity.	r. Expressed in Arc.	w_m. Mean weight of one Observation.	ρ. Probable Error of Beginning and End.	n. Number of Meteors.	Radiant. R.A. 45°+.	Radiant. Decl. 57°+.
Denning	10, 1877	8·41	±4·82	7·9	±0·68	24	−3·74±0·94	+0·74±0·54
„	10, 1878	7·61	4·36	6·3	0·62	20	+0·70±1·20	−0·63±0·49
„	9, 1880	6·63	3·80	5·9	0·54	13	+2·12±1·25	+0·61±0·57
„	9, 1893	2·58	1·48	5·0	0·21	16	−1·96±0·47	−0·21±0·23
„	10, 1896	4·40	2·52	5·3	0·36	29	+0·06±0·53	+0·52±0·29
„	11, 1898	2·72	1·56	5·7	0·22	32	+0·16±0·26	+1·15±0·20
„	10, 1899	2·18	1·25	7·2	0·21	15	−1·28±0·29	−0·34±0·18
Morine	9, 1893	6·38	3·66	5·2	0·51	23	−5·08±0·74	−1·30±0·72
„	11, 1893	3·70	2·12	5·4	0·29	26	+2·60±0·44	+1·22±0·29
Maggi	10, 1872	19·96	11·44	15·8	1·62	46	+0·45±0·96	+0·90±0·78
Scharbe	9, 1898	5·31	3·04	2·1	0·43	17	+4·01±1·52	+2·26±0·80
„	11, 1898	5·24	3·00	2·2	0·42	26	−2·22±1·28	+1·78±0·48
Lewitzky	9, 1898	7·17	4·11	8·7	0·57	11	−3·63±0·97	−0·38±0·83
„	11, 1898	8·36	4·79	6·4	0·67	12	−3·85±1·32	+0·64±1·00

The fourth column gives the value of the probable error of p in arc of great circle; if a small circle is drawn round the radiant with this value of r as radius, the prolongation of a meteor's path backwards to the radiant would be just as likely to pass without this circle as within, if the observation were of weight unity. If this is divided by the square root of w_o, we get the mean radius of this circle for each observer. It will be noticed that the mean weight of one of Maggi's observations is more than twice that of any of the other observers; the reason is that the former registers his meteor paths as much longer than most observers. But in spite of their great weight his observations have rather a large probable error, which may be accounted for by his having been influenced by the position of stars near to the meteor's path, for most of his meteors begin and end at a star. Of the 46 meteors observed by him, there are only 29 distinct paths; in one case 6 meteors are recorded as pursuing identically the same path. At the time of making these observations Maggi was a practised observer, having recorded paths for several years previously. Morine had observed at Poulkova for two or three years before; but I do not know how far back Lewitzky and Scharbe's observations extend. Mr. Denning seems to have become more skilled with practice, and his probable error is now very small.

But it must be remembered that there is mixed up with the probable error here found a possible real diffuseness of the radiant, and it is therefore desirable to discuss observations of other showers; but it is not easy to find a large enough number of meteor-paths all observed on *one* night to give a reliable probable error. However, these observations of Mr. Denning's seem to bear out his belief that radiant areas as such do not really exist (*Monthly Notices*, vol. xlv. p. 96), or at any rate that the radiant area is very small, which is what one would rather expect.

That the large difference between Morine's probable error on August 9 and 11 of the same year is possibly due to real differences of radiant is confirmed by the observation of another Poulkova observer, Stratonof; a chart of the meteors shows it at once. Mr. Denning's observations on the first of these nights do not confirm the Poulkova observers, his probable error being very small.

If, then, the radiant of the *Perseids* has a very small area of diffuseness, these observations show that a first-class observer is as likely as not to be in error by a quarter of a degree in recording the beginning or end of a meteor's path, whilst an observer of less extensive experience may err by half a degree. The resulting radiants will have probable errors of rather less than a quarter of a degree and of nearly three-quarters respectively.

In the opinion of Mr. Denning, three meteors will often give as good and certain a radiant as 30, especially when they are well-observed paths from a radiant of low altitude. Let us suppose then that we have 4 *Perseids* each of weight 6 and one in each quadrant and that they are inclined at 45° to the rectangular

axes, so that x and y are equally well determined. They furnish the following equations :—

$$\begin{aligned} \cdot 71x + \cdot 71y - p_1 &= 0. \quad \text{Wt. } 6. \\ \cdot 71x - \cdot 71y - p_2 &= 0. \quad 6. \\ -\cdot 71x + \cdot 71y - p_3 &= 0. \quad 6. \\ -\cdot 71x - \cdot 71y - p_4 &= 0. \quad 6. \end{aligned}$$

Whence the weights of x and y are both 12 ; if we take the probable error, r, of one observation of weight unity as $\pm 6{\cdot}0$ we have

$$r_x = \frac{r}{\sqrt{p_x}} = \pm 1{\cdot}73 \qquad r_y = \frac{r}{\sqrt{p_y}} = \pm 1{\cdot}73$$

and therefore the probable error of the deduced radiant is

$\pm 1°{\cdot}8$ in R.A. and $\pm 1°{\cdot}0$ in Declination.

A first-class observer might have $r = \pm 2{\cdot}5$, in which case the probable errors would be

$\pm 0°{\cdot}8$ in R.A. and $\pm 0°{\cdot}4$ in Declination.

But it will only be very rarely that four meteors can be observed as conveniently situated as those four ideal ones, and thus one coordinate is sure to be more strongly determined than the other. And, again, if the radiant is a point and the observer's r is known from previous observation, the more meteors that are made use of the greater will be the weight of x and y and the smaller the resulting probable error of the radiant. It seems, then, that to get a reliable determination of a radiant, as many meteors as possible should be observed and used in the reduction of their radiant point.

I have not yet been able to reduce the observations made on other days than August 9, 10, and 11, in order to discuss the motion of the radiant. The mean position of the radiants of all the observers on those three days are :—

	R.A.	Declination.	Longitude.	Latitude.
Aug. 9	$42°{\cdot}77 \pm 0°{\cdot}34$	$+56°{\cdot}94 \pm 0°{\cdot}19$	$59°{\cdot}25 \pm 0°{\cdot}24$	$38°{\cdot}42 \pm 3°{\cdot}19$
10	$44{\cdot}01 \pm 0{\cdot}23$	$+56{\cdot}94 \pm 0{\cdot}14$	$60{\cdot}05 \pm 0{\cdot}16$	$38{\cdot}15 \pm 0{\cdot}14$
11	$45{\cdot}58 \pm 0{\cdot}22$	$+58{\cdot}22 \pm 0{\cdot}15$	$61\,67 \pm 0{\cdot}15$	$39{\cdot}07 \pm 0{\cdot}15$

Summary of Results.

1. In deducing the radiant point, attention ought to be paid to the weights of the individual meteors.

2. An expression is therefore found, in a form capable of practical application, for assigning the proper weights.

3. It seems reasonable to assume that all meteors belonging to the same shower have nearly the same *real* length of path; the apparent length varies with the distance from the radiant. Since then an inordinately great or small weight—about 5 times or $\frac{1}{5}$ the mean weight for a given observer—indicates that the path of the meteor is not of a length proportionate to its distance from the radiant, it follows that the weights are to a certain extent a criterion of a meteor's right to be included in the determination of the radiant. But Mr. Denning's words, "radiant points should be determined by the observer of the meteors," must be remembered; the calculator must only include meteors which the observer gives him labelled as *Perseids*.

4. The effect of zenith attraction, rotation, and orbital motion of the Earth on one night's observations may be neglected as insensible in the case of the *Perseids*.

5. Assuming the radiant of this shower to be a point—

A. The probable error of a recorded beginning or end of a meteor's path varies so far as direction of path is concerned from $\pm 0°{\cdot}2$ to $\pm 1°{\cdot}6$.

B. To get a reliable determination of the radiant the more meteors made use of the better.

C. The probable error of a radiant derived from about 20 to 30 meteors varies from $\pm 0°{\cdot}2$ to $\pm 1°{\cdot}0$.

6. Assuming the radiant to be an area—

A. There are indications of a change in size of this area.

7. A discussion of observations of meteors belonging to other strong showers would decide the question of how much of the probable error here found is due to error of observation, and how much to real diffuseness of radiant.

8. The position of a radiant point derived from Mr. Denning's observations may be relied upon to within three-quarters of a degree of great circle, and I think that the present discussion, so far as it goes, makes it clear that eye-observations are of sufficient accuracy to show the existence of stationary radiants. Very great care must be taken in combining meteors to form a radiant, and if there is any doubt as to the reality of stationary radiants it is on this point, and not on the accuracy of the record of path, that it rests.

Leonids observed at Cambridge Observatory, 1900 November 13, 14, 15. By J. C. W. Herschel, B.A.

(Communicated by Sir Robert Ball.)

[The arrangement for observing the *Leonids* in 1900 was entrusted to Mr. J. C. W. Herschel, Research Student at the Observatory. He was assisted by the following members of the

University, to whom our obligations are due: Messrs. A. G. Whitehouse, B.A., Queens' College, L. N. G. Filon, King's College, J. A. Hubback, King's College, P. M. Marples, B.A., Jesus College, H. D. Wakely, St. John's College, and H. A. Webb, Trinity College.—ROBERT S. BALL.]

There were two objects in view: to keep a continuous count of all meteors seen per quarter hour; to record as many of these as were sufficiently well observed. The latter object practically covered the former. The track of a meteor was marked down *at once* on a blue map with needle-hole stars illuminated from behind, as described by Mr. Hinks in his paper on the Leonids of 1899 (*Monthly Notices*, April 1900). It is to the use of these maps that I attribute the compactness of the radiant areas, for the tracks are recorded in the most concise manner—a single line—in the form and place in which they are required and used throughout. Errors of intermediate steps are avoided, such as plotting down from notes.

Schiaparelli's geometrical method of combining tracks for the deduction of radiant points was used.

In the following table the second line includes those only "seen" but not "plotted"; the Leonids in the third line have already been counted under "plotted," and although a small number of those only "seen" were noted as "probably Leonids," they may be treated as a whole as belonging to other radiants.

Nov. 13. Period from	13^h	$\frac{1}{4}$	14^h	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	15^h	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	16^h	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	17^h	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	Total.
Plotted.	1	4	2	4	1	1	3	2	1	1	3	4	4	3	6	4	2	...	46
Seen.	1	4	...	...	...	...	...	1	1	...	2	2	4	1	4	...	4	...	24
Leonids.	1	2	1	2	...	...	1	1	...	...*	1	3	...	1	4	...	...	...	17

* Cloudy 10 minutes. Hazy all night with light clouds scattered about.

Nov. 14. Period from	$12\frac{1}{2}^h$	$\frac{3}{4}$	13^h	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	14^h	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	15^h	Total
Plotted.	2	2	6	10	5	3	1	...	4	...	2	35
Seen.	...	5	5	6	7	4	...	1	...	...	...	28
Leonids.	...	...	2	3	1	...	1	...	2	...	...	9

After 14^h clouds, getting gradually thicker.

Nov. 15 Period from	$11\frac{1}{4}^h$	$\frac{1}{2}$	$\frac{3}{4}$	12^h	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	13^h	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	14^h	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	15^h	Total.
Plotted.	2	4	4	6	3	3	3	4	4	3	5	...	5	3	1	1	51
Seen.	2	3	...	2	4	3	2	6	4	3	2	4	1	1	...	...	37
Leonids.	1	2	...	1	...	2	2	3	4	1	1	...	2	1	1	1	22

Misty till 11^h, clear till 15^h, afterwards thick clouds.

These figures show that the display of Leonids was very weak this year, with a slight access of strength on the 14th between $13\frac{1}{4}^h$ and $13\frac{1}{2}^h$, and a maximum on the 15th at about 13^h. It has been pointed out to me by Professor A. S. Herschel that the activity of neighbouring showers in several instances seems to fluctuate in sympathy with the chief shower. These numbers confirm this.

Now as to the radiant points deduced.

All the meteors which were plotted as coming from within and close round about the Sickle were considered *Leonids*, and only these, 48 in number, are here discussed. They were in general green with broad green streaks, though some were noted as "reddish," "yellow" or "bluish."

On the 13th 46 meteors were plotted, of which 17 were Leonids.

9 meteors definitely showed the radiants I. and VI.

7 ,, supported the radiants II., III., IV., V.

1 meteor was not assigned.

This last meteor was an outlying one, and though its line passed only 2° outside radiant I., to take count of it would much weaken the very definite determination of this radiant point, as will be seen below. (See table, columns 5 and 6.)

The determination of radiant V. is illustrative. The above seven meteors formed a somewhat loose radiant area at about 151°+29°; but when combined with the other two nights' observations, it turned out that the support of five of them was illusory—at least they fitted other radiants better. The two that remained crossed each other a little way below the mean centre of the seven, and fortunately three meteors were seen on the 15th which admirably supported the lowered position, and led me to re-assign one of the five to this radiant.

Radiants I. and VI. were active only on this night.

On the 14th 35 meteors were plotted, of which 9 were Leonids.

7 meteors definitely showed radiants II. and IV.

2 ,, supported radiants III. and VI.

On the 15th 51 meteors were plotted, of which 22 were Leonids.

19 meteors definitely showed the radiants II., III., IV., VII.

3 ,, supported radiant V. of the 13th.

Radiant VII. was active only on this night.

The following table sums up the results.

No.	Name.	Rad. α	Point. δ	Number of ↓'s. 13th	14th	15th	Total.	Mean deviation.	Diam. of includg. circle.	Nearest Radiants in Denning's Catalogue.
I.	ξ Leonids	141°·8	+17°·3	4	0	0	4	0°·7	1°·1	CIX. 4.
II.	μ Leonids	143·4	+25·3	2	3	4	9	0·8	1·5	CVIII. 6; CXV. 18, 19.
III.		147·5	+22·4	1	1	4	6	0·5	2·3	CXV. 5, 6, 12, 97 *et al.*
IV.	Great	150·5	+15·5	1	4	6	11	0·8	4·0	CXV. 56, 100.
V.	Leonid	151·9	+27·8	3	0	3	6	0·5	1·8	CXV. 68, 71.
VI.	Shower	153·0	+20·3	5	1	0	6	0·6	2·4	CXV. 80, 102, 103 *et al.*
VII.	α Leonids	156·6	+18·7	0	0	5	5	0·2	0 8	CXV. 21, 61; CXX. 7.
	Unassigned			1	0	0	1	2° from radiant I.		
				17	9	22	48			

Columns 6 and 7 give a measure of the reliability of the observations. Were all these meteors plotted on one map their lines of direction would cross each other over the whole Sickle, and it would not be very unreasonable to treat them as rough observations, and find an average central point for the whole lot. But they are worth more trouble. Closer study has shown that these 48 meteors, *which are all the Leonids that were plotted*, belong to the seven distinct and definite radiant points enumerated above. With three exceptions (both about 2°) the direction of every meteor passes within $1\frac{1}{2}°$ of one of these radiant points; only 6 exceed the degree; and the average deviation is only about $\frac{2}{3}°$. The tracks have not been altered in any way, and no theoretical corrections have been applied. This result is more defined than could have been expected.

I wish to draw attention to the concordance in place of radiants III., IV. and VI. with the radiants CXV., 97, 100, 102 in Mr. Denning's Catalogue, recorded there as having been noted, by Professor Brackett's observing party at Claremont, California, on 1898 November 12, 15, and 17, at $147\frac{1}{2}+22$, $150+17\frac{1}{2}$, $152\frac{1}{2}+22$. Such a concordance has, I think, additional weight because the three radiants were in each case noted as distinct by a single party of observers; the corroboration is not merely one found by selecting single places noted independently of each other by different observers with sensible variation of position.

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On a Method of Reducing Occultations of Stars by the Moon; together with the Reduction of Occultations observed on Three Occasions at the Liverpool Observatory. By H. C. Plummer, M.A.

The present paper is the result of an undertaking to reduce three series of observations of occultations and to render them available for the discussion of the Moon's semi-diameter. As I was not acquainted with a method which did not seem to leave something to be desired in point of clearness or convenience, I adopted one which offers certain considerable advantages. This method seems to differ essentially from those which have been previously employed, and it therefore appears desirable to describe it in some detail. The first object of the theory is to obtain a quite general equation expressing the geometrical conditions, on the assumption that the figure of the Moon is a sphere and that the line joining the observer to the star is a tangent line to the Moon at the time of the observation.

Let (x, y, z) be the position of the observer; (X, Y, Z) of the centre of the Moon; and (l, m, n) direction cosines of the star, referred to rectangular geocentric axes. Let R be the radius of the Moon. Then

$$\frac{\xi-x}{l}=\frac{\eta-y}{m}=\frac{\zeta-z}{n}$$

$$\frac{\xi-x}{\mathrm{X}-x}=\frac{\eta-y}{\mathrm{Y}-y}=\frac{\zeta-z}{\mathrm{Z}-z}$$

represent the lines joining the observer to the star and to the centre of the Moon respectively. If, now, ψ is the angle between these lines,

$$l(\mathrm{X}-x)+m(\mathrm{Y}-y)+n(\mathrm{Z}-z)=\cos\psi\,.\,\sqrt{\{(\mathrm{X}-x)^2+(\mathrm{Y}-y)^2+(\mathrm{Z}-z)^2\}}\;;$$

$$\mathrm{R}=\sin\psi\,.\,\sqrt{\{(\mathrm{X}-x)^2+(\mathrm{Y}-y)^2+(\mathrm{Z}-z)^2\}}\;;$$

$$\therefore\{l(\mathrm{X}-x)+m(\mathrm{Y}-y)+n(\mathrm{Z}-z)\}^2+\mathrm{R}^2=(\mathrm{X}-x)^2+(\mathrm{Y}-y)^2+(\mathrm{Z}-z)^2.$$

Now let ρ be the Earth's radius and ϕ the geocentric latitude at the place of observation; T, the sidereal time (or R.A. of the zenith); A, D, the apparent R.A. and declination of the star; α, δ, the geocentric R.A. and declination of the Moon. Also let r, P, s be the Moon's radius vector, parallax, and semi-diameter respectively. Then, if the axis of x has been taken through the First Point of Aries and the axis of z through the Pole,

$$\mathrm{X}=r\cos\delta\cos\alpha\;;\;\mathrm{Y}=r\cos\delta\sin\alpha\;;\;\mathrm{Z}=r\sin\delta\;;$$

$$l=\cos\mathrm{D}\cos\mathrm{A}\;;\;m=\cos\mathrm{D}\sin\mathrm{A}\;;\;n=\sin\mathrm{D}\;;$$

$$x=\rho\cos\phi\cos\mathrm{T}\;;\;y=\rho\cos\phi\sin\mathrm{T}\;;\;z=\rho\sin\phi.$$

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Making these substitutions in the above equation, we obtain

$$\begin{aligned}R^2+\{r[\cos\delta\cos D\cos(A-\alpha)+\sin D\sin\delta]\\ -\rho[\cos D\cos\phi\cos(A-T)+\sin D\sin\phi]\}^2\\ =r^2+\rho^2-2r\rho\{\cos\delta\cos\phi\cos(\alpha-T)+\sin\delta\sin\phi\}.\end{aligned}$$

Now

$$\cos\delta\cos D\cos(A-\alpha)+\sin D\sin\delta=\cos\Delta \quad \ldots \quad (1)$$

$$\cos D\cos\phi\cos(A-T)+\sin D\sin\phi=\cos Z \quad \ldots \quad (2)$$

$$\cos\delta\cos\phi\cos(\alpha-T)+\sin\delta\sin-\phi=\cos Z' \quad \ldots \quad (3)$$

where Δ is the geocentric distance of the star from the Moon's centre, Z is the true Z.D. of the star and Z′ is the true Z.D. of the Moon's centre. Hence

$$R^2+(r\cos\Delta-\rho\cos Z)^2=r^2+\rho^2-2r\rho\cos Z'$$

or
$$R^2=r^2\sin^2\Delta+\rho^2\sin^2 Z-2r\rho(\cos Z'-\cos\Delta\cos Z).$$

But $r\sin P=1$; $r\sin s=R$; the Earth's equatorial radius being unity.

Therefore

$$\sin^2 s=\sin^2\Delta+\rho^2\sin^2 P\sin^2 Z-2\rho\sin P(\cos Z'-\cos\Delta\cos Z) \quad (4)$$

and this is the geometrical equation sought.

We are now in a position to derive the equations of condition. Since ρ and P do not occur independently in the fundamental equation, p may be written for $\rho\sin P$. Also put S equal to $\sin^2 s$, and let

$$C=\cos Z'-\cos\Delta\cos Z \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

$$\Sigma=\sin^2\Delta+p^2\sin^2 Z-2pC \quad \ldots \quad \ldots \quad (6)$$

Then, if the observed and tabular quantities involved were quite accurate, the equation $S=\Sigma$ would be satisfied exactly. Actually, however, the quantities s, p, α, δ, A, D, and t, the mean time of the observation, require corrections ds, dp, $d\alpha$, $d\delta$, dA, dD, and dt. Substituting $s+ds$ for s, &c., an equation is obtained of the form :

$$\Sigma-S=\sin 2s\,.\,ds-Bdp-Ed\alpha-E'dA-Fd\delta-F'dD-Hdt \quad (7)$$

wherein

$$E=\frac{\partial\Sigma}{\partial\alpha};\ F=\frac{\partial\Sigma}{\partial\delta};\ E'=\frac{\partial\Sigma}{\partial A};\ F'=\frac{\partial\Sigma}{\partial D};$$

$$\begin{aligned}H=\frac{d\Sigma}{dt}&=\frac{\partial\Sigma}{\partial T}\cdot\frac{dT}{dt}+\frac{\partial\Sigma}{\partial\alpha}\cdot\frac{d\alpha}{dt}+\frac{\partial\Sigma}{\partial\delta}\cdot\frac{d\delta}{dt}\\ &=G+E\frac{d\alpha}{dt}+F\frac{d\delta}{dt} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)\end{aligned}$$

and

$$B=\frac{\partial\Sigma}{\partial p}=2(p\sin^2 Z-C) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

Hence

$$E=-2\cos\Delta\frac{\partial}{\partial a}(\cos\Delta)-2p\left\{\frac{\partial}{\partial a}(\cos Z')-\cos Z\frac{\partial}{\partial a}(\cos\Delta)\right\}$$

$$=-2\{(\cos\Delta-p\cos Z)\cos\delta\cos D\sin(A-a) -p\cos\delta\cos\phi\sin(a-T)\}\ldots(10)$$

$$F=-2\cos\Delta\frac{\partial}{\partial\delta}(\cos\Delta)-2p\left\{\frac{\partial}{\partial\delta}(\cos Z')-\cos Z\frac{\partial}{\partial\delta}(\cos\Delta)\right\}$$

$$=-2\{(\cos\Delta-p\cos Z)[-\sin\delta\cos D\cos(A-a)+\sin D\cos\delta] +p[-\sin\delta\cos\phi\cos(a-T)+\cos\delta\sin\phi]\}\ \ldots(11)$$

and

$$G=\frac{\partial\Sigma}{\partial T}\cdot\frac{dT}{dt}=\frac{dT}{dt}\cdot\left\{-2p^2\cos Z\frac{\partial}{\partial T}(\cos Z) -2p\left[\frac{\partial}{\partial T}(\cos Z')-\cos\Delta\frac{\partial}{\partial T}(\cos Z)\right]\right\}$$

$$=-2p\frac{dT}{dt}\{(p\cos Z-\cos\Delta)\cos D\cos\phi\sin(A-T) +\cos\delta\cos\phi\sin(a-T)\} \quad \ldots \quad (12)$$

Finally,

$$E'=-2\cos\Delta\frac{\partial}{\partial A}(\cos\Delta)+2p\left\{\cos\Delta\frac{\partial}{\partial A}(\cos Z) +\cos Z\frac{\partial}{\partial A}(\cos\Delta)\right\}$$

$$=2\{(\cos\Delta-p\cos Z)\cos\delta\cos D\sin(A-a) -p\cos\Delta\cos D\cos\phi\sin(A-T)\}\ldots(13)$$

and

$$F'=-2\cos\Delta\frac{\partial}{\partial D}(\cos\Delta)+2p\left\{\cos\Delta\frac{\partial}{\partial D}(\cos Z) +\cos Z\frac{\partial}{\partial D}(\cos\Delta)\right\}$$

$$=2\{(\cos\Delta-p\cos Z)[\cos\delta\sin D\cos(A-a)-\cos D\sin\delta] +p\cos\Delta[-\sin D\cos\phi\cos(A-T)+\cos D\sin\phi]\}\ldots(14)$$

We thus have the means of calculating accurately all the coefficients of the equation of condition. It is only necessary to add, since p is not itself a tabular quantity, that

$$dp=\rho\cos P\,.\,dP+\sin P\,.\,d\rho \quad \ldots \quad \ldots \quad (15)$$

It is now necessary to reconsider the above formulæ from the point of view of the computer. In the first place cos Z and cos Z′ can be found by means of (2) and (3). It is assumed that 7-figure logarithms are used, and it may be pointed out that perfect accuracy will be attained if the terms of Σ, none of which in fact exceeds ·0005, are calculated to ten places of decimals. Equation (1) is unsuitable for obtaining $\sin^2\Delta$, but from it can be derived

$$\begin{aligned}\sin^2\Delta &= 1-\cos^2\Delta = 1-\{\cos(D-\delta)-2\cos\delta\cos D\sin^2\tfrac{1}{2}(A-a)\}^2\\ &= \sin^2(\delta-D)+4\cos\delta\cos D\cos(\delta-D)\sin^2\tfrac{1}{2}(A-a)\\ &\quad -4\cos^2\delta\cos^2 D\sin^4\tfrac{1}{2}(A-a)\\ &= \sin^2(\delta-D)+\cos D\cos\delta\cos(\delta-D)\sin^2(A-a)\\ &\quad +\sin 2D\sin 2\delta\sin^4\tfrac{1}{2}(A-a) \quad \ldots \quad \ldots \quad \ldots \quad (16)\end{aligned}$$

In this form the last term is generally negligible, but in any case it is easily added as a small correction; its importance depends only on the fact that it is always positive. From $\sin^2\Delta$ can be deduced cos Δ, and then we have C from (5). This is the most convenient way of calculating C, but it must be said that the resulting value of $2p$C cannot be trustworthy beyond nine places, since C is known only to seven places of decimals. This falls short of the highest standard of accuracy, but I have generally contented myself with this method: partly because of its convenience, partly because the error introduced is an *accidental* one, small in comparison with those which depend on the star's place and the observed time, and partly because other methods failed to reveal errors in the simpler process, carefully performed, such as seemed to necessitate or warrant the extra labour involved by their adoption. Two alternatives may, however, be mentioned here. The angles Z and Z′ are known from their cosines. Then (5) may be written

$$C = 2\sin\tfrac{1}{2}(Z+Z')\sin\tfrac{1}{2}(Z-Z')+2\cos Z\sin^2\tfrac{1}{2}\Delta \quad \ldots \quad (17)$$

Or referring to the *geocentric* spherical triangle formed by the zenith (Z), the Moon's centre (M) and the star (S), it is seen that

$$C=\cos Z'-\cos\Delta\cos Z=\sin\Delta\sin Z\cos ZSM.$$

But if P is the pole, $ZSM=PSM+ZSP=\varpi+\zeta$, say.

$$\left.\begin{aligned}&\therefore\ C=\sin\Delta\sin Z\cos(\varpi+\zeta)\\ &\text{where } \sin\varpi=\cos\delta\sin(A-a)/\sin\Delta\\ &\qquad \sin\zeta=\cos\phi\sin(T-A)/\sin Z\end{aligned}\right\}\ldots \quad \ldots \quad (18)$$

These latter equations may not give the auxiliary angles without ambiguity: in that case reference to a figure will remove the difficulty. The use of equations (18) involves considerable extra labour, but they possess the advantage that a comparison of the

value of C so obtained with the value obtained directly from (5) furnishes a very complete if not a very accurate check on the work. There is no further difficulty in finding $\Sigma - S$, which is at present expressed in circular measure.

In finding the coefficients of equation (7) 5-figure logarithms are of course amply sufficient. In my own computations, B has been found from equation (9). In finding E, F and G the following slightly modified approximate formulæ have been used :—

$$E = -2 \cos\delta \{(1 - p \cos Z) \cos D \sin(A - \alpha) - p \cos\phi \sin(\alpha - T)\} \quad \ldots \quad (19)$$

$$F = -2 \{(1 - p \cos Z) \sin(D - \delta) + p[\sin\phi \cos\delta - \sin\delta \cos\phi \cos(T - \alpha)]\} \quad \ldots \quad (20)$$

$$G = 2p \frac{dT}{dt} \cos\phi \cos D \{\sin(A - \alpha) \cos(A - T) - p \cos Z \sin(A - T)\} \quad \ldots \quad (21)$$

In equation (8) $\frac{d\alpha}{dt}$ and $\frac{d\delta}{dt}$ are most easily expressed in seconds of arc per second of mean time; for the sake of uniformity therefore we take $\log\left(2\frac{dT}{dt}\right) = 1{\cdot}47831$. It is easy to see, either by comparing equations (10) and (13), and (11) and (14), or from general considerations, that $E' = -E$ and $F' = -F$ approximately. In consequence of this it has not been thought worth while to calculate the coefficients E′ and F′ separately. In fact these coefficients are not needed with great accuracy, for if considerable values be assigned to dA and dD, terms of the second order will become necessary in equation (7). The term in $d\rho$ is most convenient if we are dealing with observations made at a single observatory. If we have to combine observations made at different observatories, it will be necessary to remove the term in $d\rho$ in equation (15) and to use $-B\rho \cos P$ (which differs little from $-B$) as the coefficient of a term in dP on the right-hand side of equation (7). If it is desired to improve ρ by adding $d\rho$, the quantity $B \sin P\, d\rho$ must be added to the left-hand side of the same equation. For an increase of altitude of 209 feet, $d\rho = {\cdot}00001$, while B sin P is not likely to exceed ·0001. In the final equation we divide $\Sigma - S$ by sin 1″ in order to transform the residual from circular measure into seconds of arc, and divide the whole equation by sin $2s$ so as to make the coefficient of ds unity.

It is well known that refraction exercises an effect, which is of a parallactic nature, on the circumstances of an occultation. If there were no terrestrial atmosphere it would be necessary for the observer to change his position in order to see the occultation at the same time that he makes his observation under actual conditions. The effect, which is generally evanescent and always small, is presumably neglected in the Greenwich reductions, since

no reference is made to it, but it is taken into account by those who follow Bessel, as for instance by Küstner in his admirable memoir on "Nine Occultations of the *Pleiades*" It is easy to see that the effect is allowed for, if the radius ρ is increased by an amount x such that

$$1+x=\mu \sin z/\sin Z$$

where μ is the refractive index of the air at the surface of the Earth, and z is the apparent Z.D. of the star (*cf. Chauvenet's Astronomy*, 5th ed., vol. i., p. 516). Now $z=Z-a \tan Z$ where $a \tan Z$ is small. Consequently

$$1+x=\mu(\sin Z-a \sin Z)/\sin Z=\mu(1-a)$$

or
$$x=\mu-1-\mu a=\mu-1-a.$$

It is quite clear that an accurate calculation of this quantity can be made only if μ and a are known with great exactness. If we adopt Gladstone and Dale's law as to the dependence of the refractive index on pressure, and Mascart's numbers (*Comptes Rendus*, t. lxxviii., p. 617) for the index itself and the temperature coefficient, we have

$$\mu-1=\cdot 0002923(1-\cdot 00383t)\frac{p}{p_0}.$$

And using Bessel's formula and numbers for the refraction we have

$$a=\alpha\beta^{\lambda}\gamma^{\lambda}=\alpha\left(\frac{p}{p_0}\right)^{1+m}(1-\epsilon t)^{1+n}$$

where m and n are small, and $\epsilon=\cdot 00364$. At the zenith, when $t=0°$C and $p=760$ mm., $\alpha=\cdot 0002927$, we may put then $\alpha=\cdot 0002927-a'$ for any Z.D., and let $p/p_0=1+q$. If we make these transformations in the expression for x, we obtain a formula which is certainly erroneous in a slight degree, inasmuch as it gives a negative value of x at the zenith when $q=t=0$. This may be due to the fact that Bessel's temperature coefficient ·00364 is too small. However it is only necessary to retain terms amounting to ·000001, and from an examination in detail it is concluded that we may take $x=a'(1+q-\cdot 00364t)$, and that no serious error will be introduced by assuming, as has been generally done, a mean state of the atmosphere, in which case we find the following :—

Z=	0°	50°	60°	65°	70°	75°	80°	85°
x=	·000 000	000	001	002	003	005	012	038

It is to be feared that the constants of refraction may be insufficiently exact in the only case where they are of importance here, *i.e.* at great zenith distances. For this reason the correction has not been applied in the reductions which follow, but its

magnitude in the few instances in which it amounts to 0″·01 may be recorded here, though, as a matter of fact, in no case has it any importance. In the final equations the coefficient of $dp = -B/\sin 2s = B'$ say, and the residual is $(\Sigma - S)/\sin 2s \sin 1''$. It is only necessary then to add $B \sin P \,.\, x/\sin 2s \sin 1'' = -B'Px$, where P is expressed in seconds, to correct the final equation for the effect of refraction. The equations affected are confined to Series I, and the corrections are −0″·02 in No. 3, −0″·01 in Nos. 5, 6, 7, and 9, and +0″·01 in No. 11.

The accompanying reductions refer to observations made by the Director of the Liverpool Observatory on the following dates :—

1897 July 23	(*Monthly Notices*,	vol. lviii.,	p. 20)
1898 Dec. 27	(„	„ lix.,	p. 165)
1899 Dec. 16	(„	„ lx.,	p. 222)

On the first of these dates the *Pleiades* were occulted, and on the other two the Moon was eclipsed. A few corrections have been made in the designations of the stars, and in several instances it has not been found possible to identify the stars occulted. In Series I the mean places of the stars have been taken from Elkin's memoir on the *Pleiades*, except in those cases where the apparent places were given in the *Nautical Almanac* of the year. In columns 5 and 6 are to be found apparent coordinates (A and D) as used in the reduction. The stars in Series II are all taken directly from a list prepared by the photographic method at the Poulkova Observatory (*Astr. Nachr.*, No. 3533). For Series III the three stars B.D. +22°, Nos. 999, 1004 and 1006, were taken from the *Astr. Ges.* Catalogue (Berlin). The coordinates of No. 1011 are the result of my own reduction of a plate taken at the University Observatory, Oxford, by Mr. Bellamy, to whom, as well as to Professor Turner, I am indebted for kind assistance in this connection. The corrections deduced from the plate to the Berlin meridian places are these :—

	$\Delta A = -$ s	$\Delta D = -$ ″
B.D. +22°, 999	0·05	0·4
1004	+0·02	−0·2
1006	+0·02	+0·2

As regards the places of the Moon (α and δ), columns 7 and 8 contain the values deduced by interpolation from the *Nautical Almanac* with the following assumed corrections added :—

	$\Delta\alpha = +$ s	$\Delta\delta = -$ ″
1897 July 23	0·06	0·0
1898 Dec. 27	+0·20	+2·3
1899 Dec. 16	+0·38	+0·8

These are based on observations made at Greenwich, in the third case directly, and in the two former by estimation, not very reliable probably, from the results of previous and subsequent nights. The unpublished results were kindly communicated to me by the Astronomer Royal. The assumed position of the observatory is that given in the *Nautical Almanac*, and no correction has been made for the altitude, which is about ·000011 in units of the Earth's radius. The omission contributes to the term dp, and might be allowed for in the way already explained. But this is unnecessary if the term is eliminated from the final equations, and if this is done the result will be independent of the assumed figure of the Earth.

I. Occultation of the Pleiades, 1897 July 23.

Reference No.	Name of Star.	Phase.	Sidereal Time. h m s	Star's R.A. h m s	Star's Decl. ° ′ ″	Moon's R.A. h m s	Moon's Decl. ° ′ ″	Moon's Parallax. ′ ″	Moon's Semi-diameter. ′ ″
1	Electra	D.	20 17 23·16	3 38 47·72	23 47 32·1	3 35 41·67	24 22 54·3	54 11·72	14 47·53
2	8	,,	20 52 17·98	3 40 8·60	23 52 38·45	3 36 55·60	24 26 22·0	54 11·82	14 47·56
3	Merope	,,	20 52 56·59	3 40 14·96	23 37 48·3	3 36 56·96	24 26 25·8	54 11·82	14 47·56
4	10	,,	21 1 50·90	3 40 21·90	23 56 14·3	3 37 15·83	24 27 18·4	54 11·84	14 47·57
5	18	,,	21 16 45·59	3 41 14·14	23 49 23·5	3 37 47·43	24 28 46·2	54 11·88	14 47·58
6	*p*	,,	21 17 31·11	3 41 15·21	23 48 3·3	3 37 49·04	24 28 50·7	54 11·89	14 47·58
7	Alcyone	,,	21 21 28·16	3 41 23·92	23 47 22·1	3 37 57·42	24 29 13·8	54 11·90	14 47·58
8	29	,,	21 54 28·77	3 42 24·22	24 1 55·0	3 39 7·43	24 32 26 6	54 11·99	14 47·60
9	Pleione	,,	22 13 9·53	3 43 5·70	23 49 29·1	3 39 47·08	24 34 14·5	54 12·05	14 47·62
10	32	,,	22 17 48·19	3 43 15·64	24 4 9·4	3 39 56·94	24 34 41·3	54 12·07	14 47·63
11	9	R.	21 47 45·87	3 40 10·84	23 52 18·45	3 38 53·18	24 31 47·4	54 11·97	14 47·60
12	Alcyone	,,	22 15 46·66	3 41 23·92	23 47 22·1	3 39 52·64	24 34 29·6	54 12·06	14 47·62

II. Occultations during the Lunar Eclipse, 1898 December 27.

Reference No.	Name of Star.	Phase.	Sidereal Time. h m s	Star's R.A. h m s	Star's Decl. ° ′ ″	Moon's R.A. h m s	Moon's Decl. ° ′ ″	Moon's Parallax. ′ ″	Moon's Semi-diameter. ′ ″
1	B.D. + 23° 1392	D.	4 59 30·11	6 26 53·84	23 20 35·7	6 25 38·52	23 34 28·4	54 6·78	14 46·19
2	B.D. + 23 1402	,,	5 3 32·66	6 27 42·12	23 7 57·0	6 25 47·17	23 34 12·0	54 6·76	14 46·18
3	B.D. + 23 1403	,,	5 13 6·43	6 27 57·08	23 7 29·3	6 26 7·61	23 33 33·2	54 6·70	14 46·16

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Reference No.	Name of Star.	Phase.	Sidereal Time.	Star's R.A.	Star's Decl.	Moon's R.A.	Moon's Decl.	Moon's Parallax.	Moon's Semi-diameter.
			h m s	h m s	° ′ ″	h m s	° ′ ″	′ ″	′ ″
4	B.D. + 22° 1385	D.	5 37 32·64	6 28 4·28	22 51 36·1	6 26 59·90	23 31 53·0	54 6·54	14 46·12
5	B.D. + 23 1407	,,	5 58 24·96	6 28 31·15	23 16 40·0	6 27 44·53	23 30 26·5	54 6·41	14 46·08
6	B.D. + 22 1392	,,	6 0 7·44	6 29 3·42	22 56 19·6	6 27 48·18	23 30 19·4	54 6·40	14 46·08
7	B.D. + 23 1415	,,	6 26 55·14	6 29 51·71	23 1 56·0	6 28 45·44	23 28 27·3	54 6·23	14 46·04
8	B.D. + 23 1416	,,	6 27 40·36	6 29 53·23	23 0 58·3	6 28 47·05	23 28 24·1	54 6·22	14 46·04
9	B.D. + 22 1369	R.	5 26 21·10	6 26 27·11	22 54 31·8	6 26 35·97	23 32 39 0	54 6·61	14 46·14
10	B.D. + 23 1389	,,	5 39 13·62	6 26 35·70	23 11 28·6	6 27 3·50	23 31 46·1	54 6·53	14 46·12
11	B.D. + 23 1407	,,	6 20 9·13	6 28 31·15	23 16 40 0	6 28 30·98	23 28 55·7	54 6·27	14 46·05
12	B.D. + 23 1402	,,	6 23 12·12	6 27 42·12	23 7 57·0	6 28 37·50	23 28 42·9	54 6·25	14 46 04
13	B.D. + 22 1385	,,	6 32 30·15	6 28 4·28	22 51 36·1	6 28 57·37	23 28 3·7	54 6·19	14 46·03
14	B.D. + 23 1403	,,	6 32 40·18	6 27 57·08	23 7 29·3	6 28 57·73	23 28 3·0	54 6·19	14 46·03

III. Occultations during the Lunar Eclipse, 1899 *December* 16.

Reference No.	Name of Star.	Phase.	Sidereal Time.	Star's R.A.	Star's Decl.	Moon's R.A.	Moon's Decl.	Moon's Parallax.	Moon's Semi-diameter.
			h m s	h m s	° ′ ″	h m s	° ′ ″	′ ″	′ ″
1	B.D. + 22° 1004	D.	6 27 28·95	5 37 1·77	22 37 38·0	5 36 43·27	22 56 30·5	56 11·31	15 20·18
2	B.D. + 22 1006	,,	6 36 36·24	5 37 9·10	22 39 8·3	5 37 4·39	22 56 15·4	56 11·11	15 20·12
3	B.D. + 22 1011	,,	6 44 25·03	5 37 45·09	22 30 33·2	5 37 22·47	22 56 2·3	56 10·95	15 20·08
4	B.D. + 22 999	R.	6 58 22·01	5 36 12·17	22 32 54·08	5 37 54·74	22 55 38·6	56 10·64	15 19·99
5	B.D. + 22 1006	,,	7 18 20·79	5 37 9·10	22 39 8·3	5 38 40·95	22 55 3·9	56 10·21	15 19·88

I. *Occultation of the Pleiades*, 1897 *July* 23.

1.	$+1''\cdot25 = ds$	$-\cdot0241\ dp$	$+\cdot7364\ d\alpha$	$+\cdot5833\ d\delta$	$+\cdot6497\ dt$
2.	$+4\cdot17 = ds$	$+\cdot0248\ dp$	$+\cdot7511\ d\alpha$	$+\cdot5680\ d\delta$	$+\cdot6403\ dt$
3.	$+2\cdot02 = ds$	$+\cdot8683\ dp$	$+\cdot8256\ d\alpha$	$-\cdot4379\ d\delta$	$+\cdot5811\ dt$
4.	$+2\cdot06 = ds$	$-\cdot1375\ dp$	$+\cdot6379\ d\alpha$	$+\cdot7121\ d\delta$	$+\cdot5861\ dt$
5.	$+7\cdot11 = ds$	$+\cdot5206\ dp$	$+\cdot9125\ d\alpha$	$+\cdot1109\ d\delta$	$+\cdot6690\ dt$
6.	$-3\cdot33 = ds$	$+\cdot5895\ dp$	$+\cdot9047\ d\alpha$	$-\cdot0007\ d\delta$	$+\cdot6520\ dt$
7.	$-2\cdot03 = ds$	$+\cdot6566\ dp$	$+\cdot9065\ d\alpha$	$-\cdot0739\ d\delta$	$+\cdot6439\ dt$
8.	$+5\cdot14 = ds$	$+\cdot1032\ dp$	$+\cdot7607\ d\alpha$	$+\cdot5634\ d\delta$	$+\cdot6043\ dt$
9.	$+0\cdot45 = ds$	$+\cdot8544\ dp$	$+\cdot8055\ d\alpha$	$-\cdot4760\ d\delta$	$+\cdot5115\ dt$
10.	$+3\cdot65 = ds$	$+\cdot2025\ dp$	$+\cdot8065\ d\alpha$	$+\cdot4623\ d\delta$	$+\cdot5996\ dt$
11.	$-5\cdot08 = ds$	$-\cdot5788\ dp$	$-\cdot9041\ d\alpha$	$-\cdot0263\ d\delta$	$-\cdot3391\ dt$
12.	$-4\cdot40 = ds$	$-\cdot0049\ dp$	$-\cdot6912\ d\alpha$	$-\cdot6449\ d\delta$	$-\cdot2701\ dt$

II. *Occultations during the Lunar Eclipse*, 1898 *December* 27.

1.	$+5\cdot63 = ds$	$-\cdot4064\ dp$	$+\cdot3122\ d\alpha$	$+\cdot9433\ d\delta$	$+\cdot0961\ dt$
2.	$+2\cdot85 = ds$	$+\cdot1580\ dp$	$+\cdot9140\ d\alpha$	$+\cdot1099\ d\delta$	$+\cdot3881\ dt$
3.	$+2\cdot20 = ds$	$+\cdot1345\ dp$	$+\cdot9130\ d\alpha$	$+\cdot1113\ d\delta$	$+\cdot3814\ dt$
4.	$+4\cdot95 = ds$	$+\cdot5078\ dp$	$+\cdot4789\ d\alpha$	$-\cdot8603\ d\delta$	$+\cdot2416\ dt$
5.	$-0\cdot80 = ds$	$-\cdot4159\ dp$	$+\cdot3996\ d\alpha$	$+\cdot8985\ d\delta$	$+\cdot1036\ dt$
6.	$+0\cdot80 = ds$	$+\cdot2968\ dp$	$+\cdot8192\ d\alpha$	$-\cdot4530\ d\delta$	$+\cdot3574\ dt$
7.	$+1\cdot64 = ds$	$-\cdot0129\ dp$	$+\cdot9181\ d\alpha$	$+\cdot0419\ d\delta$	$+\cdot3480\ dt$
8.	$+5\cdot96 = ds$	$+\cdot0157\ dp$	$+\cdot9232\ d\alpha$	$-\cdot0189\ d\delta$	$+\cdot3538\ dt$
9.	$-1\cdot21 = ds$	$+\cdot2531\ dp$	$-\cdot6465\ d\alpha$	$-\cdot7074\ d\delta$	$-\cdot4214\ dt$
10.	$-0\cdot14 = ds$	$-\cdot3473\ dp$	$-\cdot8073\ d\alpha$	$+\cdot4738\ d\delta$	$-\cdot3222\ dt$
11.	$-2\cdot38 = ds$	$-\cdot4797\ dp$	$-\cdot0493\ d\alpha$	$+\cdot9946\ d\delta$	$-\cdot0810\ dt$
12.	$-2\cdot32 = ds$	$-\cdot2240\ dp$	$-\cdot8311\ d\alpha$	$+\cdot4265\ d\delta$	$-\cdot3449\ dt$
13.	$-0\cdot52 = ds$	$+\cdot3234\ dp$	$-\cdot7184\ d\alpha$	$-\cdot6216\ d\delta$	$-\cdot2316\ dt$
14.	$+3\cdot43 = ds$	$-\cdot2108\ dp$	$-\cdot8273\ d\alpha$	$+\cdot4402\ d\delta$	$-\cdot3478\ dt$

III. *Occultations during the Lunar Eclipse*, 1899 *December* 16.

1.	$-3\cdot24 = ds$	$-\cdot4330\ dp$	$+\cdot6954\ d\alpha$	$+\cdot6501\ d\delta$	$+\cdot2544\ dt$
2.	$-2\cdot77 = ds$	$-\cdot4950\ dp$	$+\cdot5822\ d\alpha$	$+\cdot7711\ d\delta$	$+\cdot1988\ dt$
3.	$-3\cdot59 = ds$	$-\cdot2908\ dp$	$+\cdot8910\ d\alpha$	$+\cdot2406\ d\delta$	$+\cdot3431\ dt$
4.	$+0\cdot26 = ds$	$-\cdot0643\ dp$	$-\cdot7082\ d\alpha$	$+\cdot4364\ d\delta$	$-\cdot4354\ dt$
5.	$+2\cdot52 = ds$	$-\cdot1817\ dp$	$-\cdot4068\ d\alpha$	$+\cdot8982\ d\delta$	$-\cdot2492\ dt$

It was obvious, of course, that the above reductions were altogether insufficient in number to give by themselves any reliable value of the semi-diameter of the Moon, but an attempt was made to examine in a general manner the results which

might be deduced from the equations. The attempt was not successful, and it is not without interest to consider why this was so. The case of the ten disappearances of stars in the *Pleiades*, I. 1–10, is typical. The equations were given equal weights after division by the coefficients of dt, which term was afterwards neglected of course. The star places are known to be very accurate, and the residual, therefore, is fairly attributable to the error in the observed quantity—the time, and can be treated in accordance with the theory of least squares. Theoretically, this method seems reasonable, but it may have the result of giving an overwhelming weight to the result of an occultation at a point of elevation or depression in the Moon's figure. It would seem, then, that a compromise is desirable in some cases, but the question is one which I do not pretend to decide. As regards the equations referred to, this difficulty did not exist, and the four normal equations were at once formed in ds, dp, da and $d\delta$. It was then found that these equations were not, in any real sense, consistent and independent, as would be necessary for the determination of the four unknowns. In fact, they could be reduced to two without sacrificing any of the information which they were capable of giving. This result can be referred, I think, to two simple causes. In the first place, the observations do not extend over a long interval of time, and the Moon's change of position is small. Now in any position it is clear that if a compensating change were made in the radius of the Moon, the Moon's distance from the observer might be changed without any effect on the phenomena. Consequently the Moon's *apparent* semi-diameter and parallax cannot be determined independently from occultations observed during a short time at a single observatory. In a similar way if disappearances alone are considered, it is evident that the effect of moving the centre of the Moon along its own apparent path while changing at the same time its semi-diameter so that the advancing point of the edge retains its position, will be very small in the case of occulted stars which lie near that path. In fact, in both cases we shall be trying to determine small quantities of the second order, and it is inevitable that the results will be expressed in terms of the residual errors. The difficulty can only be overcome by combining results obtained at distant observatories, and it is in the hope that the reductions given above may be utilised in this manner that this work has been undertaken and published.

Oxford: 1901 *January* 7.

Observations of Occultations of Stars and Saturn by the Moon, made at the Royal Observatory, Greenwich, in the Year 1900.

(*Communicated by the Astronomer-Royal.*)

Day. 1900.		Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
Jan.	9	Disapp. 27 Arietis	Sheepshanks Equat.	55	Dark	9 15 2·38	AC.
	9 (*a*)	,, ,,	28-inch Equat.	670	,,	9 15 2·38	B.
Feb.	6	,, δ Arietis	,,	670	,,	8 35 20·94	B.
	6	,, ,,	Sheepshanks Equat.	55	,,	8 35 20·71	WB.
	6	,, ,,	Corbett Equat.	100	,,	8 35 20·34	HF.
	6	,, ,,	Old Altazimuth	100	,,	8 35 21·05	S.
	6	Reapp. ,,	,, ,,	100	Bright	9 25 29·35	S.
	7 (*b*)	Disapp. A^2 Tauri	Sheepshanks Equat.	55	Dark	7 2 25·77	R.
April	4 (*c*)	,, *o* Tauri	Astrographic Equat.	225	,,	9 29 18·84	H.
	4	,, ,,	Sheepshanks Equat.	55	,,	9 29 19·86	R.
May	7	,, 19 Sextantis	Old Altazimuth	100	,,	10 42 2·20	S.
	15	Reapp. 15 Ophiuchi	Sheepshanks Equat.	100	,,	13 58 22·55	B.
June	2	Disapp. κ Cancri	Old Altazimuth	100	,,	8 31 35·36	H.
July	11	,, ξ^2 Sagittarii	Sheepshanks Equat.	55	,,	12 18 44·25	AC.
	11	,, ,,	Astrographic Equat.	225	,,	12 18 44·24	S.

Day. 1900.		Phenomenon.			Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
Sept.	3 (*d*)	Disapp.	Outer Edge of Saturn's Ring, 1st Limb		28-inch Equat.	470	Dark	7 14 59·55	L.
	3	,,			Sheepshanks Equat.	120	,,	7 15 (9·10)	AC.
	3 (*e*)	,,			Corbett Equat.	100	,,	7 15 2·55	B.
	3	,,			Astrographic Equat.	225	,,	7 15 1·29	R.
	3	,,			Old Altazimuth	100	,,	7 15 (8·54)	S.
	3	,,	Inner Edge of Saturn's Ring, 1st Limb		28-inch Equat.	470	,,	7 15 17·21	L.
	3	,,			Corbett Equat.	100	,,	7 15 16·01	B.
	3	,,			Astrographic Equat.	225	,,	7 15 16·25	R.
	3	,,			Old Altazimuth	100	,,	7 15 (21·00)	S.
	3	,,	Saturn's Ball, 1st Limb		28-inch Equat.	470	,,	7 15 24·48	L.
	3	,,	,,	,,	Sheepshanks Equat.	120	,,	7 15 27·04	AC.
	3	,,	,,	,,	Corbett Equat.	100	,,	7 15 24·98	B.
	3	,,	,,	,,	Astrographic Equat.	225	,,	7 15 26·22	R.
	3	,,	,,	,,	Old Altazimuth	100	,,	7 15 (28·97)	S.
	3	,,	,,	2nd Limb	28-inch Equat.	470	,,	7 16 24·02	L.
	3	,,	,,	,,	Sheepshanks Equat.	120	,,	7 16 (19·89)	AC.
	3	,,	,,	,,	Corbett Equat.	100	,,	7 16 23·82	B.
	3	,,	,,	,,	Astrographic Equat.	225	,,	7 16 23·56	R.
	3	,,	,,	,,	Old Altazimuth	100	,,	7 16 (28·80)	S.

Day. 1900.	Phenomenon.		Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
Sept. 3	Disapp.	Inner Edge of Saturn's Ring, 2nd Limb	28-inch Equat.	470	Dark	7 16 34·60	L.
3	,,		Sheepshanks Equat.	120	,,	7 16 35·85	AC.
3 (*e*)	,,		Corbett Equat.	100	,,	7 16 38·78	B.
3	,,		Astrographic Equat.	225	,,	7 16 (44·01)	R.
3	,,		Old Altazimuth	100	,,	7 16 (42·76)	S.
3	,,	Outer Edge of Saturn's Ring, 2nd Limb	28-inch Equat.	470	,,	7 16 57·23	L.
3	,,		Sheepshanks Equat.	120	,,	7 16 56·79	AC.
3	,,		Corbett Equat.	100	,,	7 16 56·73	B.
3	,,		Astrographic Equat.	225	,,	7 16 57·37	R.
3	,,		Old Altazimuth	100	,,	7 16 57·22	S.
3 (*f*)	Reapp.	Inner Edge of Saturn's Ring, 1st Limb	Astrographic Equat.	120	Bright	8 10 10·39	R.
3	,,	Saturn's Ball, 1st Limb	,, ,,	120	,,	8 10 20·36	R.
3	,,	,, 2nd Limb	28-inch Equat.	470	,,	8 10 47·33	L.
3	,,	,, ,,	Astrographic Equat.	120	,,	8 10 52·27	R.
3	,,	Inner Edge of Saturn's Ring, 2nd Limb	28-inch Equat.	470	,,	8 10 59·79	L.
3 (*f*)	,,		Astrographic Equat.	120	,,	8 11 4·24	R.
3	,,	Outer Edge of Saturn's Ring, 2nd Limb	28-inch Equat.	470	,,	8 11 14·26	L.
3	,,		Astrographic Equat.	120	,,	8 11 16·21	R.
3	,,		Old Altazimuth	100	,,	8 11 16·96	S.
4	Disapp.	ξ^1 Sagittarii	28-inch Equat.	670	Dark	7 35 0·88	B.
4	,,	,,	Sheepshanks Equat.	120	,,	7 35 0·25	HF.
4	,,	,,	Astrographic Equat.	225	,,	7 35 0·93	S.

LEEDS PHILOSOPHICAL

Day. 1900.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
Sept. 4 (*e*)	Reapp. ξ' Sagittarii	28-inch Equat.	670	Bright	8 49 17·12	B.
4 (*g*)	,, ,,	Sheepshanks Equat.	120	,,	8 49 (27·81)	HF.
4	,, ,,	Astrographic Equat.	225	,,	8 49 15·69	S.
12	Disapp. π Arietis	,, ,,	225	,,	12 35 28·64	PM.
13	,, 13 Tauri	,, ,,	225	,,	9 42 44·36	S.
13	,, 14 Tauri	,, ,,	225	,,	10 14 12·30	S.
13 (*g*)	Reapp. ,,	,, ,,	225	Dark	11 8 (56·47)	S.
Oct. 3	Disapp. B.A.C. 7063	Sheepshanks Equat.	120	,,	8 53 43·85	AC.
3	,, ,,	Astrographic Equat.	225	,,	8 53 42·90	S.
8 (*g*)	Reapp. W.B.I. 209	Sheepshanks Equat.	120	,,	13 19 (20·74)	H.

Notes.

The apertures of these instruments are as follows: 28-inch Equatorial, 28 inches; Astrographic Equatorial (Guiding Telescope), 10 inches; Sheepshanks, 6·7 inches; Corbett, 6·5 inches; Old Altazimuth, 4 inches.

(*a*) Instantaneous.

(*b*) Misty. Star faded gradually.

(*c*) The Moon's dark limb was easily visible. The star appeared projected within limb before disappearance.

(*d*) Occultation of Saturn. The planet was very tremulous before the occultation, but definition improved somewhat during disappearance. The times noted are those of tangency of the elliptical outlines of the ring and ball with the limb of the Moon. The times enclosed within brackets are presumably erroneous.

(*e*) The observer noted 'Probably late.'

(*f*) Cloudy. Saturn very faint and unsteady. Not considered a good observation.

(*g*) The time noted was considered uncertain, and is evidently erroneous.

The initials L., H., AC., B., WB., R., HF., S., PM., are those of Mr. Lewis, Mr. Hollis, Mr. Crommelin, Mr. Bryant, Mr. Bowyer, Mr. Rendell, Mr. Furner, Mr. Showell, and Mr. Melotte respectively.

Royal Observatory, Greenwich:
1901 *January* 11.

Observations of the Solar Eclipse of 1900 *May* 28, *made at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer-Royal.*)

Owing to clouds the first contact could not be observed. There were, however, frequent clear intervals during the eclipse, and observations of limbs and cusps in Azimuth and Zenith Distance were made with the New Altazimuth (1) in Azimuth 70° West, (2) in the Prime Vertical, as follows :—

Azimuth 70° *W.*

Day.	Mean Solar Time.	Object.	Observed Az. or Z.D.	Secs. of Tab. A.Z. or Z.D.	Error of Tab. Az. or Z.D.	Observer.
	h m s		° ′ ″	″	″	
May 28	3 9 20·64	N. Cusp Azimuth	Az. 69 51 24·87	Az. 24·63	−0·24	HF.
	3 9 53 53	N. Cusp Z.D.	Z.D. 47 50 18·95	Z.D. 18·73	−0·22	,,
	3 11 12·63	S. Cusp Azimuth	Az. 69 50 48·28	Az. 48·29	+0·01	,,
	3 11 46·08	S. Cusp Z.D.	Z.D. 48 15 53·18	Z.D. 55·48	+2·30	,,
	3 12 42·06	⊙ Second Limb	Az. 69 59 19·23	Az. 16·48	−2·75	,,

Prime Vertical.

	h m s		° ′ ″	″	″	
May 28	4 42 28·93	⊙ First Limb	Az. 89 59 32·60	Az. 30·17	−2·43	H.F.
	4 43 37·57	⊙ Lower Limb	Z.D. 62 18 33 09	Z.D. 33·49	+0·40	,,
	4 44 38·87	☽ First Limb	Az. 89 57 8·06	Az. 6·63	−1·43	,,
	4 45 7·78	N. Cusp Azimuth	Az. 90 1 46·21	Az. 40·44	−5 77	,,
	4 45 48·72	S. Cusp Z.D.	Z.D. 62 27 55·02	Z.D. 50 02	−5·00	,,
	4 46 11·83	S. Cusp Azimuth	Az. 90 8 7·46	Az. 5·76	−1·70	,,

The semi-diameters employed are deduced from the following mean values :—

(1) For the Sun from Auwers' value 16′ 1·″18 deduced from Greenwich meridian observations.

(2) For the Moon from L. Struve's value 15′ 32·″65 deduced from occultations during lunar eclipses.

Six photographs of the eclipse were obtained with the Dallmeyer photo-heliograph, the times of exposure being automatically recorded on a chronograph.

The following observations of the last contact were obtained :—

Phenomenon.	Telescope.	Aperture.	Power.	Mean Solar Time.	Observer.	Time from *Naut. Almanac.*
				d h m s		h m
Last contact	28-inch Equat.	20 inches	670	May 28 4 57 25·91	L.	4 57·5
,,	Old Altazimuth	4 inches	100	4 57 22·23	H.	
,,	Sheepshanks Equat.	6·7 inches	120	4 57 25·09	B.	
,,	Finder of Astrog. Equat.	$2\frac{1}{2}$ inches	12	4 57 21·2	W.	

The initials L., H., B., HF., W., are those of Mr. Lewis, Mr. Hollis, Mr. Bryant, Mr. Furner, and Mr. Witchell respectively.

Royal Observatory, Greenwich :
1901 *January* 11.

Light Curve of S Aræ (*Chandler* 6429).

By Alex. W. Roberts, D.Sc.

One of the most remarkable short-period variables in the southern hemisphere is the star

C.P.D. −49°, 10361,

recently discovered by Mr. Innes.

As soon after its discovery as possible, regular observations of it were begun at Lovedale with the $3\frac{1}{4}$-inch telescope. The earlier observations were made near the star's setting, and accordingly only isolated observations could be secured. These yielded apparently a period of a little over seven days.

When morning observations were possible, more continuous observations were got, and these indicated that the short period determined at the close of 1899 was in error.

A reduction of all the 1900 observations, over 200 in number, yields a period of

$10^h\ 50^m\ 45^s$.

With this period the observations made were grouped in twenty sets.

The following table gives the average magnitude of each set; also the magnitudes indicated by a mean continuous curve drawn through the observed places.

This mean curve is shown in Plate 6. The residuals between the observed and mean places are given in column 5, and the number of observations in each set in column 6.

The average departure of a single set of observations from the mean value is $0^m{\cdot}033$.

No.	Date C.M.T. d	h	m	Observed Mag. m	Mean Mag. m	R. m	No. of Obs.
1	1900 Jan. 1	0	44	10·37	10·39	−0·02	9
2	1	2	3	10·59	10·55	+0·04	11
3	1	2	47	10·63	10·64	−0·01	17
4	1	3	48	10·76	10·73	+0·03	15
5	1	4	33	10·73	10·79	−0·06	12
6	1	5	44	10·84	10·83	+0·01	7
7	1	6	34	10·74	10·77	−0·03	18
8	1	7	23	10·67	10·66	+0·01	11
9	1	7	50	10·50	10·51	−0·01	15
10	1	8	6	10·31	10·30	+0·01	10
11	1	8	13	10·08	10·10	−0·02	13
12	1	8	27	9·80	9·85	−0·05	15
13	1	8	44	9·66	9·57	+0·09	9

No.	Date C.M.T. d h m	Observed Mag. m	Mean Mag. m	R. m	No. of Obs.
14	1900 Jan. 1 8 52	9·52	9·55	−0·03	7
15	1 9 2	9·70	9·62	+0·08	10
16	1 9 17	9·73	9·76	−0·03	9
17	1 9 27	9·88	9·85	+0·03	6
18	1 9 53	10·02	10·05	−0·03	10
19	1 10 12	10·22	10·17	+0·05	10
20	1900 Jan. 1 10 38	10·25	10·27	−0·02	15

The marked characteristic of the light curve of *S Aræ* is its rapid decrease, immediately after maximum phase, and its consequent slow rate of variation at minimum.

Its rapid ascent to a maximum is notable enough, $0^m{\cdot}02$ every minute, but this is not abnormal considering the shortness of the star's period.

The form of the light curve from maximum to minimum, however, departs considerably from the usual type of short-period variation ; still the departure is not so complete as to disassociate this star from this class of variable stars. Indeed the explanation offered by more than one observer as to the cause of variation of this type meets in no incomplete way the variation of *S Aræ*.

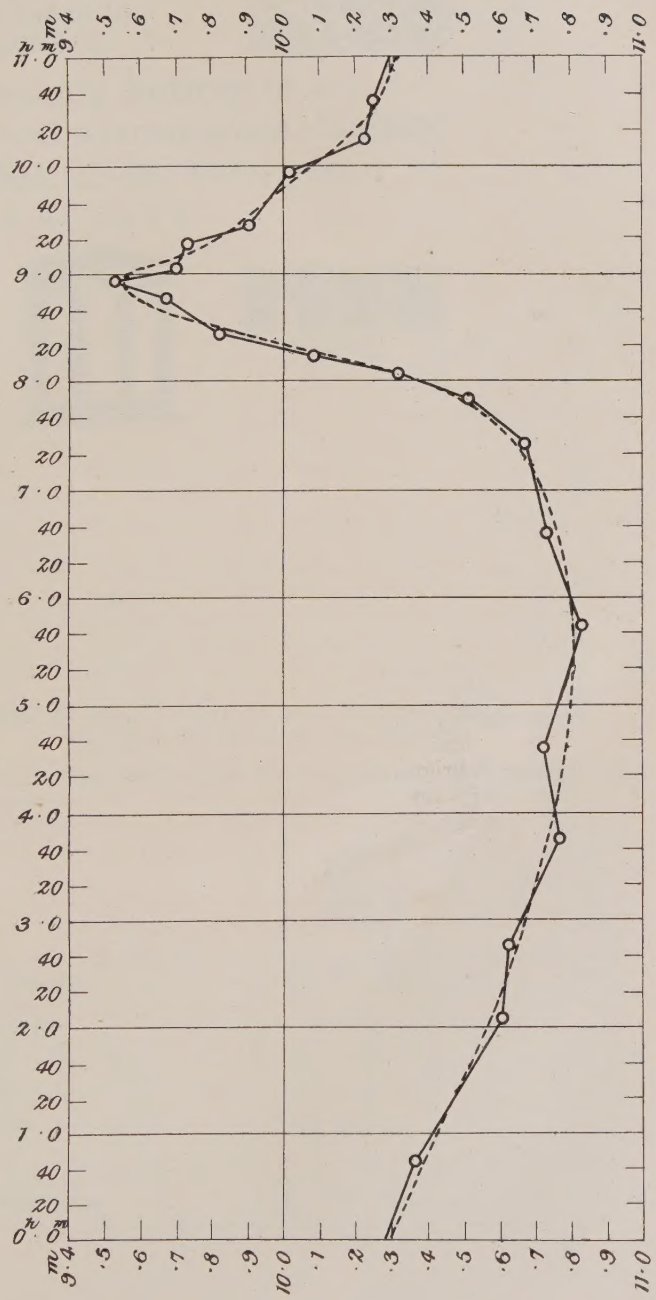
In the *Astrophysical Journal*, vol. ii. p. 283, I pointed out that if we consider short-period variables to be binary systems, with very eccentric orbits, we have all the conditions for the well-known phenomena of short-period variation : a rapid rise to maximum ; a slow descent to minimum, a descent becoming still slower as the star approaches minimum ; and continuous variation.

An eccentricity of 0·5 would yield sufficient periodic change in the mutual tidal force of each component of *S Aræ* to cause an alteration in brightness of at least one magnitude ; while even a retardation of a few minutes of the maximum tidal effect after periastron passage would produce the rapid rise to maximum brightness that is so markedly a feature of the variation of *S Aræ*, as well as of all stars of this type.

It may be of service to give in a complete form the elements of variation of *S Aræ* resulting from the Lovedale measures.

Period	$10^h\ 50^m{\cdot}45^s$
Epoch of max. ...	1900 January $1^d\ 4^h\ 12^m$ (G.M.T.)
Epoch of min.	1 7 20
Ratio of I to D	0·29
Limits of variation	$9^m{\cdot}53$ to $10^m{\cdot}84$

Lovedale : 1900 *December.*



Light curve of S Aræ. (Chandler 6423)
1900 Jan.y 1 (C.M.T.)